Index Clause Valuation under Stochastic Inflation and Interest Rate

Kamil Kladívko[1], Pavel Zimmermann[2]

Abstract. This work is focused on valuation of the reinsurer's share on a particular accident, stemming from the excess of loss (XL) reinsurance contract that applies to a general insurance annuity compensation, e.g., motor liability or workmen's compensation. The excess of loss (XL) reinsurance contract is an insurance contract between the insurer and reinsurer, which guarantees recovery payment to the insurance company for each accident in the amount of the accident that the insurance company pays off in excess of a contracted priority.

Special focus is set on the impact of the so called index clause which is usually included to the reinsurance contract. The index clause allows the reinsurer to increase the originally agreed priority by a coefficient which is, roughly said, calculated as a ratio of the sum of all nominal payments to the sum of all deflated payments.

Analytic valuation formula does not exist for the reinsurer's share without the index clause. Hence simulation model is used to perform the study. We assume a geometric Brownian motion for the inflation index and an Ornstein-Uhlenbeck process for the discount rate, where we allow the two processes to be correlated.

Keywords: Annuities, Index clause, Reinsurance, Excess of loss.

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1 Introduction

This work is focused on valuation of the reinsurer's share stemming from the excess of loss reinsurance contract. Special focus is set on the impact of a specific clause, the so called index clause on the value of the reinsurer's share. We will limit our self to one accident, which is already reported. We will first specify the type of (direct) insurance products of our concern and consequently define the reinsurer's share considered.

The above mentioned reinsurance contract applies to various *insurance products*, which are marketed by insurance companies to clients. The insurance product considered is a general insurance product that guarantees regularly inflated payment until death or a specified age of the victim (or other beneficiary). Typical example of such a product is the motor third party liability in the central and eastern Europe, where certain amount of accidents are paid out regularly in the form of an *annuity* (e.g. the compensation for loss of income or care costs in the case of large bodily accidents). Another example is the workmen's compensation, commonly existing in the western Europe. Note that "business wise" (not accounting wise) these liabilities belong to non-life insurance lines, despite the fact that the amount paid off depends on survival or death of the beneficiary. This fact implies certain complications to valuation (or modeling in general) of these liabilities:

1. These liabilities are in comparison to other non-life liabilities extremely long-termed, and hence much more sensitive to discounting or inflation.
2. These liabilities are typically reinsured with non-proportional treaty (usually the excess of loss

[1]University of Economics, Prague W. Churchill Sq. 4,130 67 Prague 3, kladivko@gmail.com
[2]University of Economics, Prague W. Churchill Sq. 4,130 67 Prague 3, zimmerp@vse.cz

treaty), which is rarely encountered in life insurance annuities and the reinsurer's share may be more significant than in life insurance liabilities.

The *reinsurance contract* is an insurance contract between the insurer and reinsurer. In this contract the reinsurer agrees in exchange for financial compensation (reinsurance premium) to share the insurer's liabilities to some extend in a form of recovery payments to the insurer. In this paper, we deal with the *excess of loss* contract (XL contract). The XL contract guarantees recovery payment to the insurance company for each accident in the amount of the accident that the insurance company pays off in excess of a contracted amount. The recovery is due at the accident settlement date. The contracted amount is referred to as the *priority.*

In case of excess of loss reinsurance contract, reinsurers often do not accept the inflation risk embedded in such liabilities, and therefore include the so called *index clause* in the reinsurance treaty. The purpose of the index clause is to adjust the priority for movements of an inflation index. The index clause allows the reinsurer to increase the originally agreed priority by a coefficient which is, roughly said, calculated as a ratio of the sum of all nominal payments to the sum of all deflated payments. Notice that the reinsurance share is calculated at the time of the claim closure and therefore past inflation as well as all payments are known. Given the length of the annuity liabilities the index clause may have significant effect on the reinsurer's share and, therefore it is particularly important to include this clause in the valuation model.

The existing literature on the index clause is quite limited. Several notes on the impact of the inflation on the nonproportional reinsurance as well as reasoning for the index clause and reinsurance pricing formulas can be found in [3]. A pricing model for an excess of loss treaty including (among other clauses) also the stability (index) clause was published in [4]. An indexing method for annual aggregate deductibles and limits was scrutinized in [5]. A method for the estimate of the insurance liabilities based on individual loss level with inclusion of the excess of loss reinsurance with the index clause was developed in [2]. Deterministic evaluation of the impact of the index clause together with its analytical properties and capitalization strategy was published in [6].

The main focus of this paper is to derive the the difference in the expected present value of the reinsurer's share on technical reserve of a reported accident using a *stochastic model.* Note that as we assume that the claim is already reported, we may also assume that we already know the age and gender of the victim as well as the intensity of the compensation.

2 Reinsurer's Share and Underlying Processes

2.1 Compensation, Annuities and the Underlying Economy

We assume that the insurance product guarantees a compensation paid out continuously with a constant intensity a, to a victim with some residual lifetime. In this subsection, only fixed residual lifetime t will be considered. The following payment streams are considered:

- $A^*(0,t) = \int_0^t a du = at$: The total payment from time 0 to time t (in a constant price level).
- $A(0,t) = \int_0^t aI(u)du$: The total payment, which is continuously adjusted by an inflation index $I(u)$, specified below, from time 0 to time t. The inflation adjusted payment $A(0,t)$ corresponds to the aggregated amount paid by insurance company to the victim.

To calculate the present value of the considered cash flows, we introduce a stochastic discount rate. Specifically, the discount rate follows an Ornstein-Uhlenbeck process

$$dr(u) = \kappa\big(\mu - r(u)\big)du + \sigma dW_1(u), \quad r(0) = r_0, \tag{1}$$

where μ an unconditional mean of the discount rate, κ determines a speed at which the short-rate reverts to its unconditional mean, and σ is the volatility of the discount rate. The inflation is represented by a consumer price index (CPI), which follows a generalized geometric Brownian motion

$$dI(u) = \pi(u)I(u)du + \gamma I(u)\big(\varrho dW_1(u) + \sqrt{1-\varrho^2}dW_2(u)\big), \quad I(0) = 1, \tag{2}$$

where $\pi(u)$ may be thought of as the time dependent expected inflation rate, and where γ is the volatility of the CPI. The CPI is exposed to shocks that drive the discount rate, and in addition to shocks that are orthogonal to the discount rate, whereas ϱ determines the correlation between the CPI and the discount rate.

To sum up, our naive economy is generated under the real-world probability measure by the two independent Brownian motions $W_1(u)$ and $W_2(u)$. In particular, we do not assume an existence of a market with financial instruments, such as nominal and real bonds, which could be used for hedging the considered insurance and reinsurance products.

2.2 Reinsurer's Share

The the two reinsurance products will now be defined. In what follows, we repeatedly use the following manipulation of the max function: for any $x \geq 0$, $(x(y-z))^+ = x(y-z)^+$.

1. *Reinsurer's share with an index clause.* At a given time t, the reinsurer's payoff is given as

$$C(t) = \left(A(0,t) - \frac{A(0,t)}{A^*(0,t)}K\right)^+ = \left(at - K\right)^+ \frac{1}{t}\int_0^t I(u)du, \tag{3}$$

   where K is the priority of the reinsurance contract, and the fraction $A(0,t)/A^*(0,t) = (1/t)\int_0^t I(u)du$ represents the index clause, which removes the inflation risk from the priority. The max part of the payoff is deterministic and thus known at time 0. The only stochastic part of the payoff is time average of the inflation process. This time average can be interpreted as an Asian forward contract on the inflation process with delivery price set to 0, or alternatively as a zero-strike arithmetic Asian call on the CPI.

2. *Reinsurer's share without an index clause.* At a given time t, the reinsurer's payoff is given as

$$W(t) = \left(A(0,t) - K\right)^+ = at\left(\frac{1}{t}\int_0^t I(u)du - \frac{K}{at}\right)^+, \tag{4}$$

   which can be interpreted as an arithmetic Asian call option on the CPI with a fixed strike equal to $K/(at)$.

2.3 Mortality Risk

To this point we have considered the reinsurer's share for a fixed residual lifetime t. We now introduce a random variable T_x, which measures a random time at which an x-year old victim dies (residual lifetime). We follow the standard assumption of independence between the mortality and economic variables, in particular the probability distribution of T_x is independent of the Brownian motions $W_1(u)$ and $W_2(u)$.

The Makeham's model is assumed for the survival function of lifetime of the population of newborns, ${}_tp_0 = P(T_0 > t)$. Namely the following survival function is assumed:

$${}_tp_0 = bs^t g^{c^t}, \tag{5}$$

where b, s, g and c are parameters. The corresponding probability density function is denoted $f_x(t)$. The (conditional) density of the residual lifetime of an individual being already x years old is then

$$f_x(t) = \frac{f_0(x+t)}{{}_xp_0} = -\frac{d}{dt}\frac{{}_{x+t}p_0}{{}_xp_0}. \tag{6}$$

2.4 Valuation of the Reinsurer's Share

The standard approach to calculating the reinsurer's share value is based on taking the expectation of the discounted reinsurer's share payoff. Therefore, based on (3) and (4) we get for a fixed nonrandom time t the respective valuation formulas:

$$\text{With IC: } C(0,t,I_0,r_0,K) = \big(at - K\big)^{+}\mathbb{E}\left[e^{-\int_0^t r(u)du}\frac{1}{t}\int_0^t I(u)du\right]. \tag{7}$$

$$\text{Without IC: } W(0,t,I_0,r_0,K) = at\mathbb{E}\left[e^{-\int_0^t r(u)du}\left(\frac{1}{t}\int_0^t I(u)du - \frac{K}{at}\right)^{+}\right]. \tag{8}$$

Note that we calculate the reinsurer's share value at time 0, i.e., we assume that the accident has just occurred. It is straightforward to show that the value of the reinsurer's share at some later time $\tau > 0$ can be expressed in terms of a newly reported accident of the same type, but with an altered priority and intensity of the payout.

Further note that the expectation operator $\mathbb{E}[\cdot]$ in the valuation formulas is taken with respect to the Brownian motions $W_1(u)$ and $W_2(u)$. Since the two Brownian motions are assumed to be independent of the mortality risk, it is straightforward to incorporate the random time T_x into the valuation. Specifically, given our mortality model, the respective valuation formulas incorporating the uncertain maturity of the reinsurer's share are as follow:

$$\text{With IC: } C(0,T_x,I_0,r_0,K) = \int_0^\infty C(0,t,I_0,r_0,K)f_x(t)dt. \tag{9}$$

$$\text{Without IC: } W(0,T_x,I_0,r_0,K) = \int_0^\infty W(0,t,I_0,r_0,K)f_x(t)dt. \tag{10}$$

Despite an enormous effort, which has produced many academic papers, a search for an analytical solution for the price of an arithmetic Asian call has not been successful yet, not even for a deterministic discounting. Therefore, we use Monte Carlo methods to evaluate the expected values of (7) and (8). However, we would like to note that we have obtained an almost analytical solution (modulo fast and accurate numerical integration) for the reinsurer's share with an index clause under the here considered setting. The solution will be presented in a subsequent paper, where also an economy with a complete financial market is introduced, and therefore a risk-neutral valuation approach to the reinsurer's share is developed.

3 Parameter Calibration

As already mentioned above, we evaluate the reinsurer's share under the real-world measure. We therefore estimate parameters of the discount rate and CPI processes (1) and (2), respectively, from the available time series data. The CPI is published by the Czech Statistical Office on a monthly basis. We adjust the CPI for seasonal effects to measure its correlation with the three-month PRIBOR, which is used as a proxy for the discount rate. For simplicity, we assume that the expected inflation rate $\pi(u)$ is a constant π. The estimated parameters, reported in the left-hand part of Table 1, are found by maximum likelihood based on monthly data from the beginning of 1998 until April 2014. The estimated parameter values are relatively stable for different sampling periods. However, the maximum likelihood estimation of the unconditional mean, μ, of the discount rate results is in positive values very close to zero, or even in negative values. This is due to an apparent downward trend in the three-month PRIBOR time series during the considered period. Since we believe that interest rates must be mean reverting, we set μ equal to 0.04.

Generation life tables constructed in [1] updated for the base values of 2007 are used. Parameters of the Makeham's function are fitted using ordinary least squares approach to the table number of survivals up to age x, denoted as l_x. (Technically, we are not fitting the survival probability but the survival function multiplied by the fixed table radix $l_0 = 100000$.) The survival probabilities are then calculated based on the formula ${}_np_0 = l_n/l_0$ and for the discrete time intervals dt, the probability of death in an interval $[t, t+dt]$ is:

$$P(t < T_x < t + dt) = \frac{{}_{x+t}p_0 - {}_{x+t+dt}\, p_0}{{}_xp_0} = \frac{l_{x+t} - l_{x+t+dt}}{l_x}. \tag{11}$$

The estimated values of the parameters are displayed in Table 1.

κ	μ	σ	π	γ	ϱ	b	s	g	c
0.30	$0.04^{*)}$	0.006	0.024	0.010	0.19	100040	0.99878	0.99997	1.12310

Table 1 The left-hand part reports the estimated parameters of the underlying economic processes. *) The unconditional mean of the discount rate has been set to 0.04 rather then estimated. The right-hand part reports the estimated parameters of the Makeham's model fitted to l_x.

4 Numerical Results

The results are organized as follows: We start with the results for a "base case", i.e., with parameters displayed in Tables 1 and 2, which are considered as a "typical example". Then we perform particular single parameter stresses to illustrate the sensitivity of the reinsurer's share on these values. All parameters, except for ϱ, are shifted up and down by 50 %. In all cases time step $dt = 1/12$ is used and 10 000 simulations were performed. Further on the parameters from Table 2 were set as 'the base case'.

a	K	x	Sex
0.5	20	30	Male

Table 2 Base case parameters. (a and K in mil. CZK.)

The results for the base case are displayed in the first row of Table 3. It is obvious that for the base case the index clause has substantial impact on the reinsurer's share, and hence on the net reserve. It is therefore essential to take this clause in consideration when evaluating reinsurance treaty or adequacy of reserves. The second line of this table displays the results when only deterministic model is applied, i.e., when both volatility parameters γ and σ are set to 0. The impact of the stochastic parameters is apparently quite low both on the values of the reinsurer's share as well as on the impact of the index clause (IC).

The first stress test is performed on the payout intensity a. It is obvious that for the down shift ($a = 250000$), the index clause annihilates the reinsurer's share. As the payout intensity increases, the reinsurer's share increases and the relative impact of the IC is decreasing. So for the highest accidents, the impact of the IC is partially mitigated.

The second stress test is performed on the inflation drift π. Although this parameter is of course significant for the value of the reinsurer's share itself, the impact on the difference between contract with and without IC is not so dramatic. For the "up shift" ($\pi = 0.036$), the increase of the relative impact of the IC is only around 6 percentage points and when π is further increased, the ratio is not increasing substantially.

The third stress test is performed on the long term mean of the nominal risk free rate μ. As the cash flows are quite long, discounting has a substantial impact on the reinsurer's share itself. But the impact on the difference between the contract with and without the IC is quite negligible.

The fourth stress test is performed on the correlation between the inflation and nominal risk free rate ϱ. Here the down shift is set to 0 (independence assumption) and the up shift is set to 0.5. It is quite surprising, that this parameter has only a very little impact on the reinsurer's share as well as the difference between the contracts with and without the IC. It is in fact quite important because the dependence assumption complicates theoretical analysis of the modeled phenomena substantially and approximation with the independent case may be a helpful benchmark.

5 Conclusions

The index clause is usually present in the excess of loss reinsurance contracts. Based on the above mentioned results, it is obvious that on reasonably large accidents, the impact of this clause on the reinsurer's share (and hence the net reserve) may be very substantial. On the other hand, stress tests indicate that although the sensitivity of the value of the reinsurer's share it self may be significant, the impact of the index clause is generally quite robust.

Stressed	Scenario	Reinsurer's share		Relative impact on the base case		Impact of IC	
		With IC	Without IC	RS with IC	RS Without IC	CZK	Relative
-	Base case	1.45	3.61	-	-	2.16	59.8%
-	Deteministic	1.44	3.58	-0.9%	-0.7%	2.14	59.9%
a	0.75	4.32	6.62	198.4%	83.4%	2.29	34.7%
[0.5]	0.25	0.00	0.80	-100.0%	-77.7%	0.80	100.0%
π	0.036	2.29	6.67	58.3%	85.0%	4.38	65.6%
[0.024]	0.012	0.96	1.77	-33.9%	-50.9%	0.81	45.9%
μ	0.06	0.48	1.27	-66.8%	-64.8%	0.79	62.2%
[0.04]	0.02	4.44	10.53	206.7%	191.8%	6.08	57.8%
ϱ	0	1.45	3.62	0.4%	0.4%	2.17	59.9%
[0.19]	0.5	1.45	3.61	0.2%	0.1%	2.16	59.8%

Table 3 Reinsurer's share and the influence of the IC for the base case and relevant ± 50 % parameter shifts. The base case value of the parameter is in [ ]. All monetary values are in mio CZK.

Additionally the sensitivity on the volatility parameters is relatively low and the difference between stochastic and deterministic approach is quite negligible. The correlation of the error term of the nominal risk free rate and the inflation index has also very little impact on the results. These results then suggest that deterministic approach would also be an acceptable alternative for valuation.

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