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Spatial variability of soil parameters in an analysis of a strip footing using hypoplastic model

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ABSTRACT: Advanced hypoplastic constitutive model is used in probabilistic analyses of a typical geotechnical problem, strip footing. Spatial variability of soil *parameters*, rather than *state variables*, is studied by means of random field Monte-Carlo simulations. The model, including correlation length, was calibrated using a comprehensive set of experimental data. Foundation displacement u_y for given load follows closely lognormal distribution, even though some model parameters are distributed normally. The vertical correlation length θ_v was found to have minor effect on $\mu[u_y]$, but significant effect on $\sigma[u_y]$, which decreases with decreasing θ_v due to spatial averaging. Applicability of a simpler probabilistic method (FOSM) is also discussed.

1 INTRODUCTION

Geomechanical properties measured in site investigation programs are highly variable. The causes for the parameter variability can be broadly divided into two groups (Helton 1997). Objective (*aleatory*) uncertainty results from inherent spatial variability of soil properties, whereas subjective (*epistemic*) uncertainty is caused by the lack of knowledge and measurement error. Both sources of uncertainty need to be considered in geotechnical design (Schweiger and Peschl 2005). In this work, we focus on the description of the aleatory uncertainty, which is inherent to the given soil deposit and cannot be reduced by additional experiments or improvement of experimental devices.

Advanced constitutive models for soils distinguish between material *parameters* and *state variables*. In principle, soil parameters are specific to the given mineralogical properties of soil particles and soil granulometry. State variables (such as void ratio e) then allow us to predict the dependency of the soil behaviour on its state. In this respect, the sources of aleatory uncertainty can further be subdivided into two groups:

1. In some situations, soil mineralogy and granulometry may be regarded as spatially invariable, and the uncertainty in the mechanical properties of soil deposit come from variability in soil state. In this case, soil *parameters* may be considered as constants, and it is sufficient to consider spatial variability of *state variables* describing relative density of soil.
2. In other cases, soil properties are variable due to varying granulometry and mineralogy of soil grains. Such a situation is for example typical for soil deposits of sedimentary basins, where the granulometry varies due to the variable geological conditions during the deposition. In such a case, it is necessary to consider spatially variable soil *parameters*.

Most of the applications of probabilistic methods in combination with advanced soil constitutive models consider uncertainty in the state variable only. As an example, Hicks and Onisiphorou (2005) studied stability of underwater sandfill berms. Their aim was to study whether presence of 'pockets' of liquifiable material may be enough to cause instability in a predominantly dilative fill. They used a double-hardening constitutive model with probabilistic distribution of state variable ψ. In other applications, Tejchman (2006) studied the influence of the fluctuation of void ratio on formation of the shear zone in the biaxial specimen using the hypoplastic model by von Wolffersdorff (1996).

The aim of the research project presented is a complete evaluation of the influence of parameter variability of an advanced constitutive model on predictions of typical geotechnical problems. Suchomel and Mašín (2009) performed a set of laboratory experiments on sandy material, that were used for evaluation of probabilistic distributions and spatial variability of parameters of a hypoplastic model for granular materials by von Wolffersdorff (1996). The influence of their variation on predictions of a typical geotechnical problem (strip footing) is presented in this contribution.

2 EXPERIMENTAL PROGRAM AND CALIBRATION OF A CONSTITUTIVE MODEL

For details of the experimental program and calibration of the models see Suchomel and Mašín (2009).

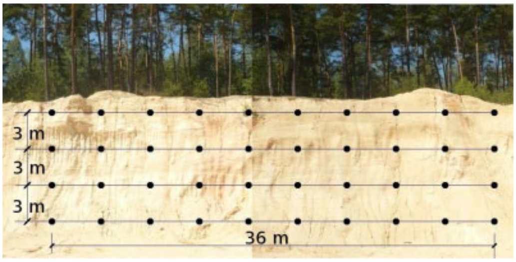

Figure 1. The wall of the sand pit in south part of the Třeboň basin. Black dots represent positions of specimens for the laboratory investigation.

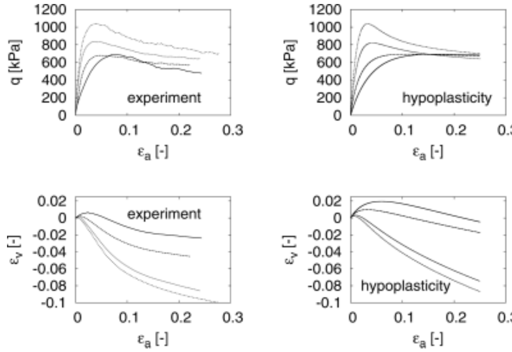

Figure 2. Typical experimental and simulated results of drained triaxial tests.

The material for investigation comes from the south part of upper Cretaceous Třeboň basin in the South Bohemia from the sand pit "Kolný". The pit is located in the upper part of the so-called Klikovské layers, youngest (senon) strata of the South Bohemian basins. These fluvial layers are characterised by a rhythmical variation of gravely sands, sands and sands with dark grey clay inclusions.

Altogether forty samples were taken from a ten meters high pit wall in a regular grid (Fig. 1). The laboratory program was selected to provide for each of the samples enough information to calibrate a hypoplastic model for granular materials by von Wolffersdorff (1996). The following tests were performed on each of the 40 samples:

- Oedometric compression test on initially very loose specimens.
- Drained triaxial compression test on specimen dynamically compacted to void ratio corresponding to the dense *in-situ* conditions. One test per specimen at the cell pressure of 200 kPa.
- Measurement of the angle of repose.

The hypoplastic model by von Wolffersdorff (1996) has eight material parameters. The model was calibrated using procedures outlined by Herle and Gudehus (1999). The whole process of calibration was automated to reduce subjectivity of calibration. Examples of the measured and simulated results of triaxial experiments are shown in Figure 2 (specimens from one column of the sampling grid).

Suitability of different statistical distributions (normal and lognormal) to represent the experimental data was evaluated using Kolmogorov-Smirnov tests. Characteristic values of statistical distributions of parameters of the hypoplastic model are given in Tab. 1 (note the results differ slightly from Suchomel and Mašín (2009), as two specimens $c1$ and $e4$ with unusual behaviour were not considered in the present evaluation). Statistical distributions of parameters h_s and β are in Figure 3, as an example.

As position of each of the 40 samples was known, Suchomel and Mašín (2009) could also evaluate the correlation length in the horizontal (θ_h) and vertical (θ_v) directions. The dependency of the correlation

Table 1. Characteristic values of statistical distributions of parameters of the hypoplastic model.

param.	dist.	mean	st. dev.
ϕ_c	log.	35.1°	1.62°
h_s	log.	3.82 GPa	14.6 GPa
n	log.	0.289	0.095
e_{c0}	norm.	0.847	0.111
e_{i0}	norm.	1.016	0.133
e_{d0}	norm.	0.318	0.042
α	log.	0.074	0.048
β	norm.	1.261	0.605

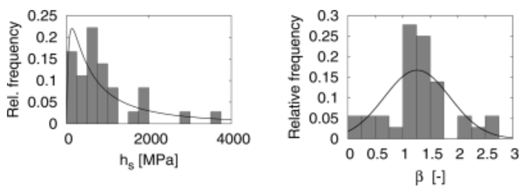

Figure 3. Examples statistical distributions of hypoplastic parameters (h_s and β).

coefficient ρ on distance was approximated using an exponential expression due to Markov

$$\rho = \exp\left[-2\sqrt{\left(\frac{\tau_h}{\theta_h}\right)^2 + \left(\frac{\tau_v}{\theta_v}\right)^2}\right] \tag{1}$$

where τ_h is the horizontal distance between two specimens and τ_v is the vertical distance. The correlation length could successfully be evaluated using parameter φ_c only. This parameters depends directly on soil granulometry. The least square fit of Eq. (1) through the experimental data is shown in Figure 4, leading to $\theta_h = 242$ m and $\theta_v = 5.1$ m. Note that practically no correlation is observed in the vertical direction, therefore the obtained value $\theta_v = 5.1$ m is implied by the adopted vertical sampling distance, rather than by the actual autocorrelation properties. Additional experiments on specimens obtained from the outcrop in a

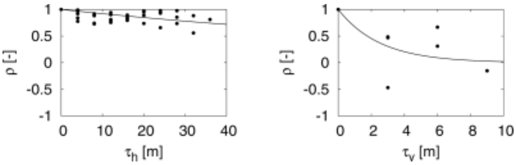

Figure 4. Evaluation of the correlation coefficient ρ in horizontal (a) and vertical (b) directions for parameter ϕ_c, together with least square fit of Eq. (1).

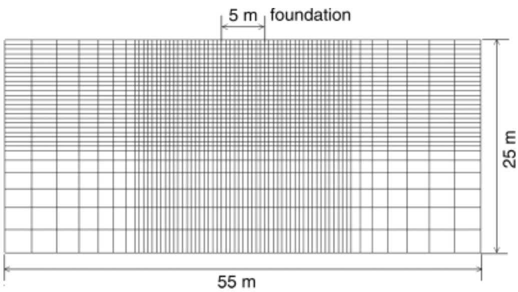

Figure 5. The problem geometry and finite element mesh.

smaller vertical sampling distance are currently being performed to evaluate θ_v more precisely.

In addition to the laboratory experiments, five *in situ* porosity tests with membrane porosimeter were performed at different locations within the area from which the samples were obtained. Average natural void ratio was 0.41. The sand was thus in a dense state.

3 STRIP FOOTING PROBLEM

The influence of spatial variation of parameters of the hypoplastic model was studied by simulations of a typical geotechnical problem – settlement of a strip footing. Simulations were performed using a finite element package *Tochnog Professional*. The problem geometry and finite element mesh are shown in Figure 5. The mesh consist of 1920 nine-noded quadrilateral elements. The foundation was analysed as rigid and perfectly smooth. Element size in the vicinity of the footing is 0.5 m.

The soil unit weight is 18.7 kN/m^3. The initial $K_0 = 0.43$ was calculated from Jáky formula $K_0 = 1 - \sin\varphi_c$, with average value of φ_c measured in the experiments. The initial value of void ratio $e = 0.48$ was used in simulations. The soil was thus slightly looser then *in situ*, in order to ensure that the void ratios do not surpass the physical lower bound e_d during Monte-Carlo simulations. Spatial variability of void ratio was not considered. The analyses thus focused on qualitative evaluation of the influence of the spatial variability of soil parameters. In all cases, foundation displacements corresponding to the load of 500 kPa were evaluated. Bearing capacity of the foundation was not evaluated, as the peak loads depend on the mesh density due to the localisation phenomena.

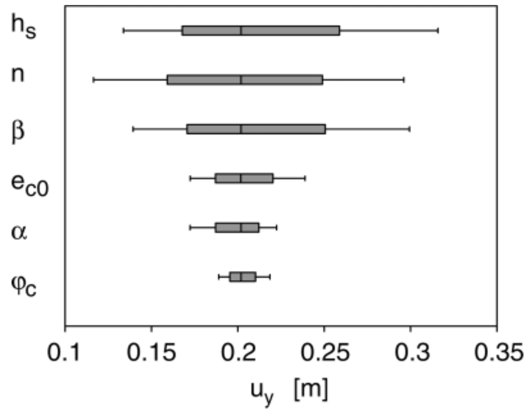

Figure 6. Tornado diagram showing sensitivity of foundation displacements on different parameters.

4 SENSITIVITY ANALYSIS

At first, sensitivity of the results on different material parameters was evaluated. In these simulations, spatial variability of soil parameters was not considered. Only one parameter was varied at a time, all other parameters were given their mean or median values (for normally and lognormally distributed parameters respectively).

A Tornado diagram showing sensitivity of foundation displacements u_y on different parameters is given in Figure 6. It shows u_y for the mean value $\mu[X]$ and for $\mu[X] \pm \sigma[X]$, where X is parameter value in the case of normally distributed parameters and its logarithm in the case of lognormally distributed parameters.

As expected, foundation settlements are influenced the most significantly by the parameters controlling soil bulk modulus (parameters h_s and n) and parameter β that influences the shear stiffness. Less significant is the influence of the relative density, controlled through parameters e_{c0}, e_{i0} and e_{d0}. Note that e_{c0} and the other two reference void ratios e_{i0} and e_{d0} were varied simultaneously to ensure constant ratios between them imposed during calibration (Suchomel and Mašín 2009). The smallest influence on foundation settlements have parameters α and φ_c, which control soil strength.

5 PROBABILISTIC ANALYSES

The following probabilistic analyses of the strip footing were performed. First of all, the problem was simulated without considering spatial variability of the parameters (i.e. the correlation length was infinite). In the second step, spatial variability of the parameters was introduced through simulations based on random field theory by Vanmarcke (1983) (RFEM). Last, applicability of a simpler probabilistic method based on Taylor series expansion (first order second moment method, FOSM) was studied.

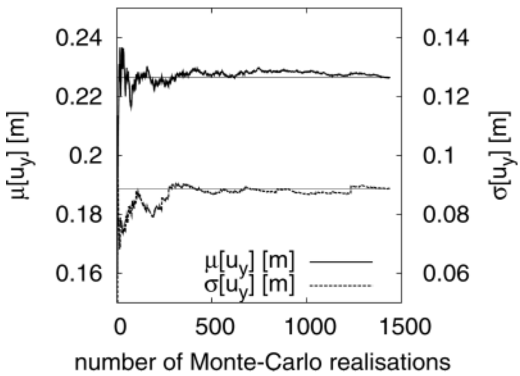

Figure 7. The dependency of $\mu[u_y]$ and $\sigma[u_y]$ on the number of Monte-Carlo realisations.

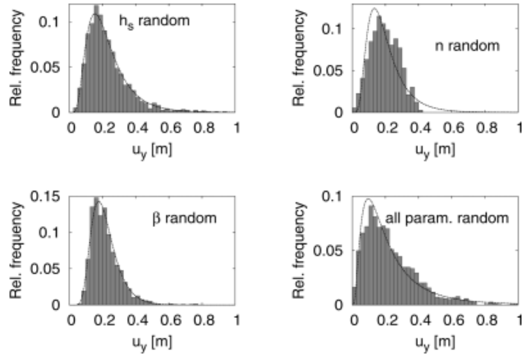

Figure 8. Probabilistic distributions of u_y for Monte-Carlo analyses with infinite correlation length.

Table 2. Results of probabilistic simulations with infinite correlation length (in meters).

random param.	RFEM		FOSM	
	$\mu[u_y]$	$\sigma[u_y]$	$\mu[u_y]$	$\sigma[u_y]$
h_s	0.231	0.128	0.193	0.107
n	0.197	0.083	0.193	0.089
β	0.217	0.087	0.193	0.077
all param.	0.229	0.163	0.193	0.164

5.1 *Simulations with infinite correlation length*

If spatial variability of the soil parameters is neglected, the problem can be simulated using approximate analytical methods (for example, FOSM method). These methods have, however, a number of limitations, as discussed in Sec. 5.3. The probabilistic aspects of the problem analysed in this contribution are fairly complex. The constitutive model and thus also the dependency of u_y on X are non-linear. Some of the model parameters follow Gaussian distribution, whereas other follow lognormal distribution. For this reason, analyses with spatially invariable fields of input variables were performed using Monte-Carlo method.

This method is fully general, but depending on the problem solved it may require significantly large number of realisations and consequently a considerable computational effort. Figure 7 shows the dependency of the mean value $\mu[u_y]$ and standard deviation $\sigma[u_y]$ for random field simulations from Sec. 5.2. At least 700 Monte-Carlo realisations is required to get a reasonably stable estimate of $\mu[u_y]$ and $\sigma[u_y]$. In all presented simulations, at least 1000 realisations were performed.

Four analyses were performed. In three of them, only one parameters was varied at a time and the other parameters were given their mean (normal parameters) or median (lognormal parameters) values. These analyses were performed for the parameters h_s, n and β. β follows normal, whereas h_s and n follow lognormal distribution. In the last analysis, all paremeters were considered as random. All parameters were simulated as uncorrelated, with the exception of e_{c0}, e_{d0} and e_{i0}, which were perfectly correlated to preserve constant ratios between them. Figure 8 shows probabilistic distributions of u_y and Tab. 2 gives the values of $\mu[u_y]$ and $\sigma[u_y]$. The distribution of the output variable is well described by the lognormal distribution, even in the case of β as single variable parameter, which itself follows the Gaussian distribution. Slight deviation from the log-normal distribution shows the analysis with n and all parameters random.

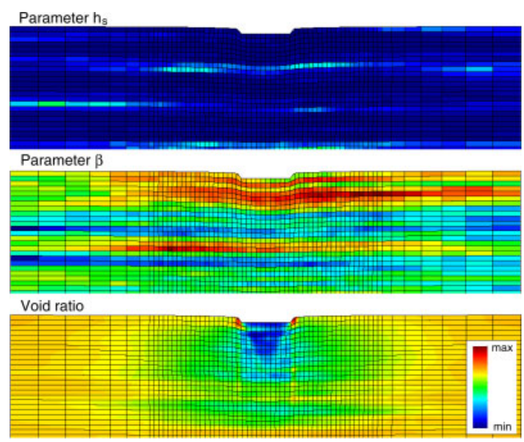

Figure 9. Typical random field simulations with $\theta_v = 5.1$ m (bottom part of the mesh not shown).

5.2 *Random field simulations*

Spatial variability of soil parameters was considered in the second set of analyses. Random fields were generated using method based on the Cholesky decomposition of the correlation matrix. Due to uncertainty in the correlation length in the vertical direction (discussed in Sec. 2), simulations were run with different values of θ_v. All parameters were considered as random, e_{c0}, e_{d0} and e_{i0} were perfectly correlated and other parameters were uncorrelated.

Example random fields (parameters h_s and β) for $\theta_v = 5.1$ m are shown in Figure 9. The same figure

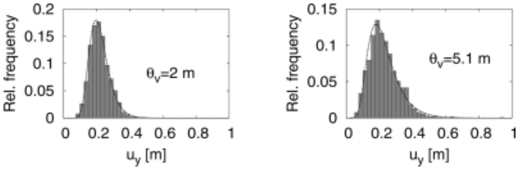

Figure 10. Probabilistic distributions of u_y in random field analyses with finite θ_v and all parameters treated as random.

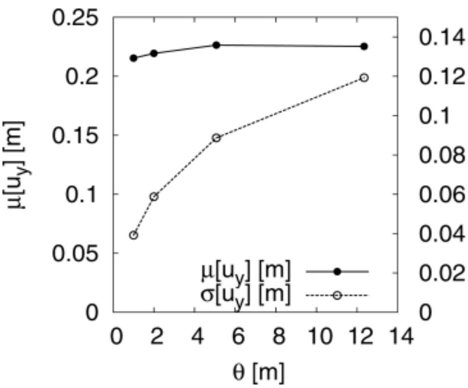

Figure 11. The dependency of $\mu[u_y]$ and $\sigma[u_y]$ on θ_v predicted by the random field method.

shows also corresponding distribution of void ratio after 0.8 m of the foundation displacement. Study of this example, as was well as other simulations not presented here, reveals that the lowest void ratios occur in softer areas characterised by low values of the parameter β. Parameter h_s, which also have a substantial influence on u_y (Sec. 4), affects due to its highly skewed lognormal distribution (Fig. 3) the results in a global way, whereas the parameter β controls the local deformation pattern.

Typical statistical distributions of the output variable u_y are shown in Figure 10. In all studied cases, u_y follows lognormal distribution. This agrees with the results of RFEM simulations with spatially invariable parameters (Sec. 5.1).

Figure 11 and Tab. 3 show $\mu[u_y]$ and $\sigma[u_y]$ predicted by the RFEM simulations with different values of θ_v. There is only slight change (decrease) of $\mu[u_y]$ with θ_v, whereas $\sigma[u_y]$ decreases with decreasing θ_v substantially. This decrease is caused by the spatial averaging of soil properties, leading to the reduction of variance of the input variables and consequently of the performance function (see Sec. 5.3 for more details).

5.3 *FOSM simulations*

One of the popular approximate analytical methods for probabilistic analyses is the first-order, second-moment (FOSM) method. Unlike the Monte-Carlo method, the FOSM method has a number of limitations. First, it consideres linear dependency of the performance function (in our case u_y) on the input variables (in our case, material parameters X). Also, it does not provide any information on the skewness of the probabilistic distribution of the output variable. Its applicability to solve the highly complex probabilistic problem from this work is studied in this section. Details of the method may be found elsewhere, see e.g. Suchomel and Mašín (2010). Parameter values (normally distributed parameters) or their logarithms (lognormal parameters) are used as an input into the FOSM method.

Table 3. Results of probabilistic simulations with variable vertical correlation length (in meters).

	RFEM		FOSM	
θ_v	$\mu[u_y]$	$\sigma[u_y]$	γ	eff. vert. dist.
1 m	0.215	0.039	0.48	1.33
2 m	0.219	0.059	0.61	1.78
5.1 m	0.226	0.089	0.75	2.53
12.3 m	0.225	0.119	0.87	3.04

Tab. 2 gives the values of $\mu[u_y]$ and $\sigma[u_y]$ by the FOSM and RFEM methods for infinite correlation length. The FOSM method underestimates both $\mu[u_y]$ and $\sigma[u_y]$ due to the non-linear dependency of the output variable u_y on the parameters h_s and β (see Fig. 6). The method does not provide any information on the skewness of the statistical distribution of u_y. Therefore, its use requires a check of the distribution of u_y through Monte-Carlo simulation (or other general probabilistic method). In our case, the distribution of u_y is clearly lognormal (Figs. 8 and 10).

As discussed by Suchomel and Mašín (2010), the FOSM method can indirectly consider spatial variability of the input variables through reduction of their variances due to spatial averaging. The reduction factor γ is defined as $\gamma = (\sigma[X_i]_A/\sigma[X_i])^2$, where $\sigma[X_i]$ describes the global statistics of the variable X_i and $\sigma[X_i]_A$ is the standard deviation of the spatially averaged field. It may be calculated by integration of the Markov function (Eq. (1)) (Vanmarcke 1983). Suchomel and Mašín (2010) have shown that in the case of a slope stability problem in spatially variable c-φ soil, γ can be estimated *a priori* by integrating the Markov function in 1D along the potential failure surface.

The value of γ may be evaluated by comparing standard deviations of the FOSM and RFEM outputs (Tab. 3). As γ is a variance reduction factor of the *input* parameters, however, a linear dependency of the ouput variable on the input parameters (or its logarithms) must be assumed. The results are thus only approximate. As expected, γ decreases with θ_v. Tab. 3 gives also an effective vertical distance below the foundation that leads to the given γ, found by 2D rectangular integration of the Markov function. This distance depends on θ_v and it thus cannot be easily estimated *a priori*. This limits applicability of the FOSM method for estimation of $\sigma[u_y]$ in the case of spatially variable parameters with finite correlation length.

6 CONCLUDING REMARKS

Advanced hypoplastic constitutive model was used in probabilistic analyses of a typical geotechnical problem, strip footing. In the analyses, spatial variability of soil *parameters*, rather than *state variables*, was emphasized. It was shown that the result are influenced the most by the soil parameters h_s, n and β. The output variable u_y was found to follow closely log-normal distribution, even in the case when normally distributed parameters (such as β) were varied. The vertical correlation length θ_v was found to have minor effect on $\mu[u_y]$, but significant effect on $\sigma[u_y]$, which decreases with decreasing θ_v due to spatial averaging. Even though the problem is highly complex and non-linear, the FOSM method was found to provide satisfactory predictions for infinite correlation length. For finite correlation length, however, the variance reduction factor γ cannot be easily estimated *a priori*.

ACKNOWLEDGEMENT

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REFERENCES

Helton, J. C. (1997). Uncertainty and sensitivity analysis in the presence of stochastic and subjective uncertainty. *Journal of Statistical Computation and Simulation 57*, 3–76.

Herle, I. and G. Gudehus (1999). Determination of parameters of a hypoplastic constitutive model from properties of grain assemblies. *Mechanics of Cohesive-Frictional Materials 4*, 461–486.

Hicks, M. A. and C. Onisiphorou (2005). Stochastic evaluation of static liquefaction in a predominantly dilative sand fill. *Géotechnique 55*(2), 123–133.

Schweiger, H. F. and G. M. Peschl (2005). Reliability analysis in geotechnics with a random set finite element method. *Computers and Geotechnics 32*, 422–435.

Suchomel, R. and D. Mašín (2009). Calibration of an advanced soil constitutive model for use in probabilistic numerical analysis. In P. et al. (Ed.), *Proc. Int. Symposium on Computational Geomechanics (ComGeo I), Juan-les-Pins, France*, pp. 265–274.

Suchomel, R. and D. Mašín (2010). Comparison of different probabilistic methods for predicting stability of a slope in spatially variable c-phi soil. *Computers and Geotechnics 37*, 132–140.

Tejchman, J. (2006). Effect of fuctuation of current void ratio on the shear zone formation in granular bodies within micro-polar hypoplasticity. *Computers and Geotechnics 33*(1), 29–46.

Vanmarcke, E. H. (1983). *Random fields: anaylisis and synthesis.* M.I.T. press, Cambridge, Mass.

von Wolffersdorff, P. A. (1996). A hypoplastic relation for granular materials with a predefned limit state surface. *Mechanics of Cohesive-Frictional Materials 1*, 251–271.