A kinematic hardening critical state model for anisotropic clays

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ABSTRACT. *The paper investigates a new approach to constitutive modelling of one–dimensionally consolidated clays. Some experimental evidence, that the state boundary surface for anisotropically consolidated clays, constructed by normalizing with respect to specific volume, has an isotropic, non–rotated shape, is presented. It has been shown, that the simple critical state models, which assume elastic behaviour inside this state boundary surface, are not capable of predicting highly non–linear soil behaviour. Predictions may be significantly improved by introducing elasto-plastic behaviour inside the state boundary surface – e.g. 3-SKH model developed by Stallebrass, 1990.*
It is shown in the paper, that the shortcoming of this constitutive model is that it is still unable to predict the direction of strain increment vector for anisotropically compressed clays, which results in an overprediction of the K_0 *stress state. In comparison with test data the model also overpredicts shear strains in triaxial compression. A modified constitutive model is presented in the paper.*
The isotropic shape of the state boundary surface is retained and an experimentally determined direction of the strain increment vector for anisotropically consolidated clays is implemented by assuming a non–associated flow rule. The modified model uses a deviatoric cross–section through the state boundary surface similar to Matsuoka–Nakai failure criterion, but retains all other features of the 3-SKH model and does not introduce any additional model parameter.
Finally the model is evaluated with respect to laboratory tests on anisotropically consolidated reconstituted clays. A clear improvement of predictions compared to the 3-SKH model is demonstrated.

1. Introduction

Critical state soil mechanics (Schofield and Wroth, 1968), led to a significant improvement of predictions of soil behaviour by introducing specific volume as an additional state variable. More recent research on the behaviour of soil in the small strain and very small strain range (e.g., Jardine et al., 1984; Stallebrass, 1990) revealed, that the assumption of elastic behaviour inside the state boundary surface is not acceptable due to the non–linearity of soil behaviour in the small strain range. A number of constitutive models have been developed to describe these phenomena and it has been shown, that such models lead to a significant improvement in finite element predictions of boundary value problems (e.g. Stallebrass and Taylor, 1997).

One possible approach to incorporating non–linearity of soil behaviour inside the state boundary surface with a critical state model is by introducing kinematic hardening (Mróz et al., 1979). This paper deals with a constitutive model developed by Stallebrass (1990) in order to incorporate the effects of recent stress history into a kinematic hardening model proposed by Al Tabbaa and Muir Wood (1989) by introducing a second kinematic 'history' surface (3-SKH model).

A modification of this constitutive model to simulate the behaviour of anisotropically consolidated clays is proposed in the paper. The experimental evidence supporting this modification is reviewed in the first part of the paper and the modified model is then described and evaluated using triaxial test data from tests on anisotropically consolidated reconstituted clays.

2. Anisotropically consolidated clays

The important aspect used to characterise soil behaviour is the shape and size of the state boundary surface. This surface is defined as a boundary of all possible states of clay in stress–specific volume space and is explicitly incorporated into critical state constitutive models. In the following, the experimentally determined shape of the state boundary surface is reviewed. Then the constitutive model, which assumes this shape of the state boundary surface and associated flow rule, is used to predict soil behaviour. Predictions are compared with experimental data to emphasize the most important differences between predicted and observed soil behaviour.

2.1. *State boundary surface*

An appropriate method to find the shape of the state boundary surface is normalization with respect to specific volume. This method is based directly on the definition of the state boundary surface. It has been shown by several authors (e.g. Pickles, 1989; Cotecchia, 1996; Cotecchia and Chandler, 2000; Rampello and Callisto, 1998), that the state boundary surface of anisotropically consolidated clays defined by this method has an isotropic, non–rotated shape.

Pickles (1989) performed a number of triaxial tests on anisotropically consolidated soft organic silty clay, with the direction of the stress paths outward the state boundary surface, thus ensuring that the stress state remains on the state boundary surface during loading. These stress paths, normalized with respect to specific volume, are shown in Fig. 1. It can be seen, that the shape of the state boundary surface is very close to the elliptical state boundary surface defined by the Modified Cam–Clay model and there is no apparent rotation of this surface in the direction of the K_0 normally consolidated (K_{0NC}) line.

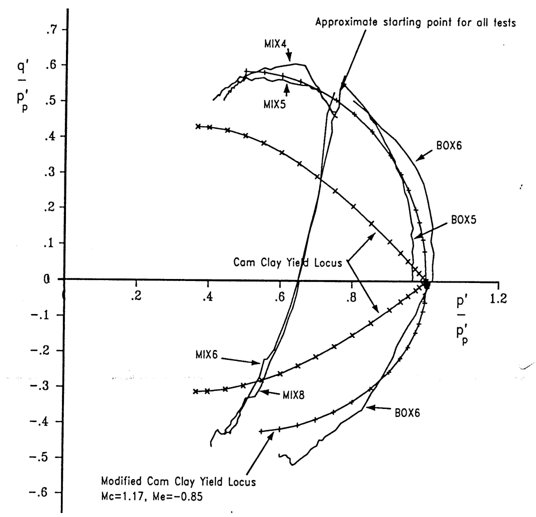

Figure 1. *Stress paths for K_0 normally compressed samples normalized with respect to specific volume (Pickles, 1989) Tests MIX4, MIX5, BOX5 and BOX6 direction toward state boundary surface, MIX6 and MIX8 undrained unloading*

On the other hand, a rotated gross yield surface, often also referred to as state boundary surface, is typically found when it is determined using the "bilinear" method. This method is based on the assumption, that the soil behaviour inside gross yield surface is elastic. A transition between "elastic" and elasto–plastic behaviour is than found by assuming the initial quazi–linear part of the stress–strain curve. As demonstrated by e.g., Tavenas et al. (1979), this shape is similar to the shape of contours of equal specific strain energy.

2.2. *Direction of plastic strain increment*

In this section an experimentally determined direction of the plastic strain increment vector will be compared with predictions by the 3-SKH model, in order to show that the associated flow rule, which this model assumes, leads to inaccurate predictions of direction of strain increment vector. The 3-SKH model uses an elliptical shape for the state boundary surface, the same as the state boundary surface for the Modified Cam–Clay model.

It is useful to start by considering the stress state under K_0 normally consolidated conditions (K_{0NC}). In this case the direction of the total strain increment vector remains fixed and determines the K_{0NC} stress state measured. A range of experimental results are available, which allow empirical relationships proposed in the literature to be studied. One of the first empirical relationships was proposed by Jáky (1944). It has been shown in many publications (e.g., Ladd et al., 1977; Mayne and Kulhawy, 1982; Ting et al., 1994; Watabe et al., 2003), that Jáky's relationship is generally applicable to fine–grained soils.

$$K_{0NC} = 1 - \sin\phi \tag{1}$$

The 3-SKH model significantly overpredits the K_{0NC} conditions due to the over–estimation of the ratio of plastic volumetric to plastic shear strain increments in triaxial compression. On the other hand it can predict K_0 unloading stress states well (Stallebrass and Taylor, 1997).

Anisotropic compression tests on reconstituted London Clay with different stress ratios, η, were performed by Richardson (1988). These tests are valuable because they can be used to study the direction of the strain increment vector for anisotropically normally consolidated clay for different stress states than the K_{0NC} stress state. An experimentally determined direction for the plastic strain increment vector and direction predicted by the Modified Cam–Clay model are shown in Figure 2. It is

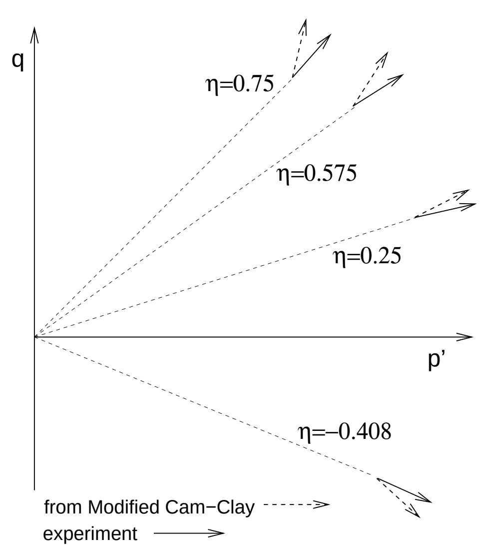

Figure 2. *Values of the total strain increment ratio predicted from the Modified Cam–Clay model and observed during anisotropic compression for reconstituted London Clay (after Richardson, 1988)*

clear that in all cases the Modified Cam–Clay model (and hence the 3-SKH model) underpredicts the ratio $\delta\epsilon_v/\delta\epsilon_s$ in triaxial compression.

Callisto (Callisto, 1996) performed a series of tests on K_0 overconsolidated specimens of reconstituted Pisa Clay. The contribution of these tests is, that it is possible to study the direction of strain increment vector for K_0 overconsolidated soils. A typical plot of the ratio $\delta\epsilon_v/\delta\epsilon_s$, where $\delta\epsilon_v/\delta\epsilon_s$ are total strain increments, with respect to q/p' for tests in triaxial compression (R30) and extension (R315) are shown in Figure 3 (the number in the test name dentotes the direction of the stress path in q/p' stress space). Stress path of the test R30 is not heading toward failure and it leads to approximatelly constant ratio of $\delta\epsilon_v/\delta\epsilon_s$ and q/p' after larger strains. Simulations

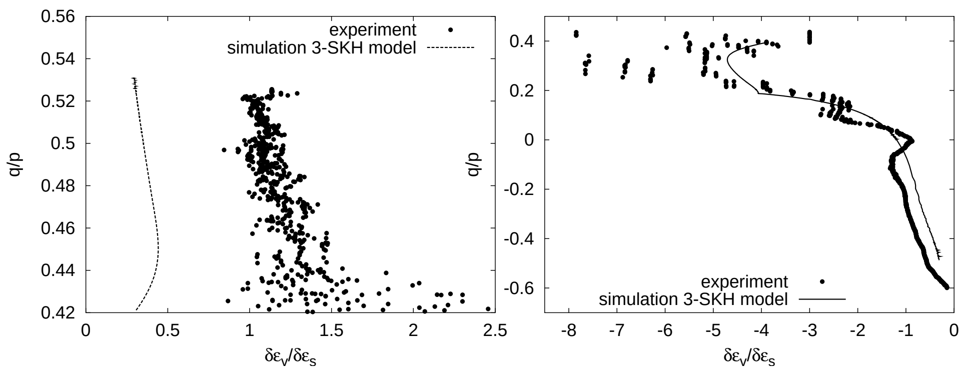

Figure 3. *$\delta\epsilon_v/\delta\epsilon_s$ – q/p plots for tests on reconstituted Pisa Clay (test data from Callisto, 1996) R30 (left) and R315 (right) and simulation by the 3-SKH model.*

by the 3-SKH model were performed using parameters derived by Baudet (Baudet, 2001). The associated flow rule leads to relatively accurate predictions of the ratio of volumetric to shear strains in triaxial extension, whereas in triaxial compression this ratio is significantly underpredicted. These results are in accordance with predictions of the K_0 stress state, which is predicted accurately for K_0 unloading, but is highly overpredicted for K_0 loading.

2.3. *Summary*

The experimental evidence shows, that the state boundary surface for anisotropically consolidated soils has non–rotated shape, provided that it has been defined by normalization with respect to specific volume. Using this state boundary surface together with a kinematic hardening constitutive model which assumes the same shape of the state boundary surface and kinematic yield surface and an associated flow rule, leads to over–estimation of the ratio of volumetric to shear strain increment in triaxial compression and relatively accurate predictions in triaxial extension. Predictions of the direction of the strain increment vector in triaxial compression may be significantly improved by assuming non–associated flow rule, as demonstrated in the following sections.

3. The 3-SKH model for anisotropic clays (AI3-SKH)

The 3-SKH model has been modified to predict the behaviour of anisotropically consolidated clays (AI3-SKH) by assuming a non–associated flow rule (Mašín, 2002). The plastic potential surface in triaxial compression has been assumed to have a more vertically elongated shape, which leads to predictions of a smaller ratio of plastic volumetric to plastic shear strain. The ratio of the major to minor axis of the plastic potential surface in triaxial compression (M_{fltc}) is calculated according to the formula given in Equation 2 such that predicted values of K_{0NC} fulfill Jáky's (1944) formula for the K_{0NC} stress state (Eqn. 1).

$$M_{fltc} = 3(6-M)\sqrt{\frac{M(1-2\nu)(\lambda-\kappa)}{\left[(1-2\nu)(6-M)\lambda - M\kappa(1+\nu)\right]\left[(6-M)^2-9\right]}} \qquad (2)$$

Therefore the model does not require an additional model parameter compared to the 3-SKH model.

The shape of the plastic potential surface in triaxial extension is kept the same as in the 3-SKH model and is therefore defined by the parameter M.

The mathematical formulation of the AI3-SKH model also includes an octahedral cross section through the state boundary surface similar to the Matsuoka and Nakai (1974) failure criterion. This ensures that the model predicts the same friction angle in triaxial compression and extension, rather than the same ratio q/p'.

4. Single element evaluation of the modified model

The AI3-SKH model has been evaluated using data from stress path controlled triaxial tests. Tests with different directions of stress path in q/p' space have been used in order to evaluate the proposed shape of the plastic potential surface.

K_0 loading and unloading tests performed by Coop et al. (1995) on reconstituted Boom Clay have been simulated using parameters for the 3-SKH model derived by Ingram (2000). Simulations by the 3-SKH and AI3-SKH model are shown in Fig. 4. It is clear, that not only the K_0 normally consolidated stress state, but also the K_0 stress state during unloading is predicted accurately by the AI3-SKH model. Predictions of K_0 unloading are similar to predictions by the 3-SKH model, when computation start from a K_{0NC} stress state calculated according to Eqn. 1.

Shear stages of tests on reconstituted Pisa Clay performed by Callisto (1996) described in the previous section have been simulated using the 3-SKH and AI3-SKH model. q/ϵ_s plots for tests R90, R60, R30 and R315 are given in Fig. 5. The AI3-

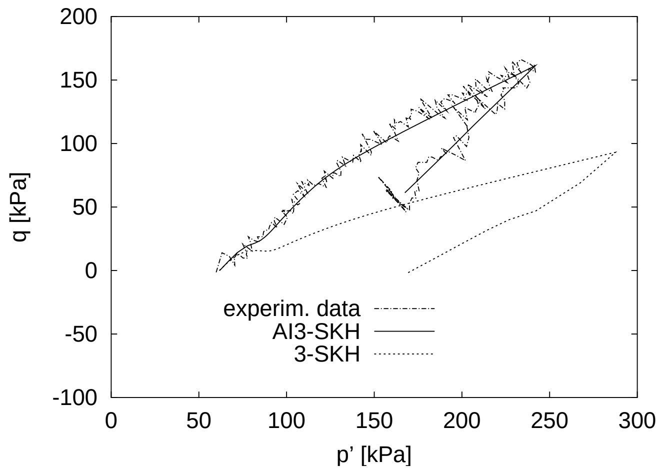

Figure 4. *Stress path of the K_0 loading and unloading test on Boom Clay (test data after Coop et al. (1995)) and simulation by the AI3-SKH and 3-SKH model.*

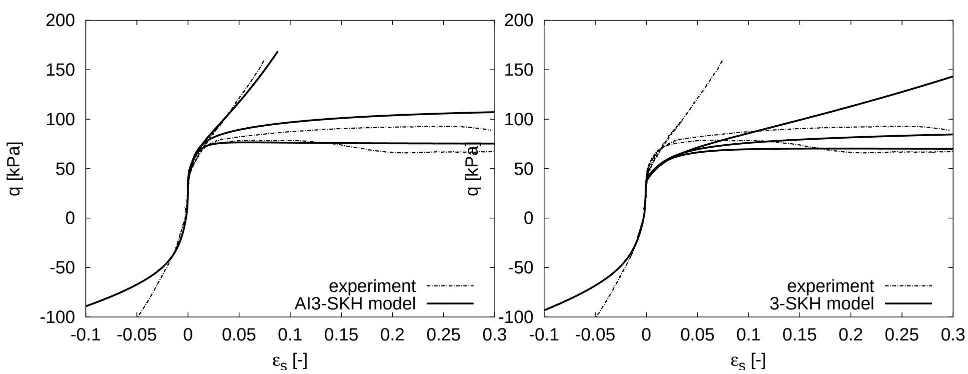

Figure 5. *q/ϵ_s plots for tests on reconstituted Pisa Clay (test data after Callisto, 1996). Simulations with the AI3-SKH model (left) and 3-SKH model (right).*

SKH model significantly improves predictions due to smaller shear strains generated. The model formulation does not influence predictions of volumetric strains, which are predicted rather accurately by the 3-SKH model (as illustrated in Fig. 6 for tests R0). It therefore also does not change the shape of the stress paths normalized with respect to specific volume.

Test data and simulations by the 3-SKH model, already presented in the section 2, are presented together with simulations by the AI3-SKH model in Figure 7. The variation in plastic strain increment ratio with stress ratio computed by the AI3-SKH model is closer to the experimental data for both tests. For test R315 (triaxial extension) both

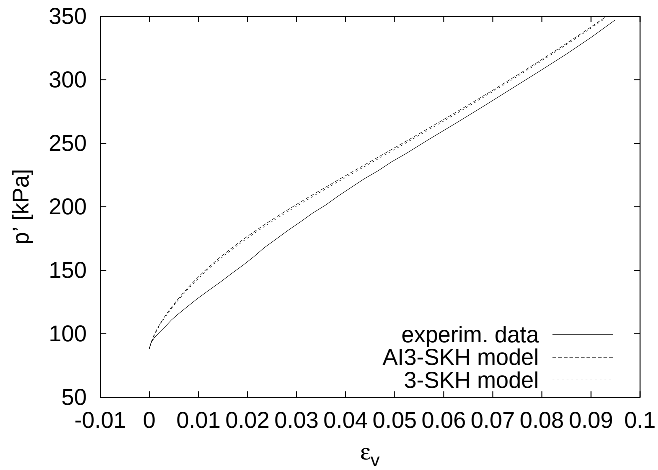

Figure 6. *p'/ϵ_v results of the test R0 (test data after Callisto, 1996) and simulations by the 3-SKH and AI3-SKH model.*

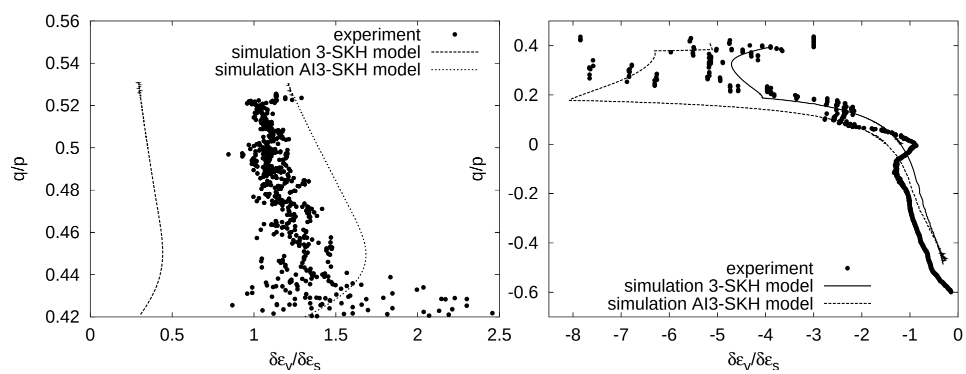

Figure 7. *$\delta\epsilon_v/\delta\epsilon_s$ – q/p plots for tests on reconstituted Pisa Clay (test data from Callisto, 1996) R30 (left) and R315 (right) and simulation by the 3-SKH model and AI3-SKH model.*

merge at large strains, whereas in triaxial compression (R30) sets of predictions are significantly different.cd /mn d da i Predictions by the AI3-SKH model are closer to the experimental data due to the larger ratio $\delta\epsilon_v/\delta\epsilon_s$ predicted. The improvement in predictions shown in Figure 4 is due to the change in the flow rule in the new model.

Contours of the equal specific strain energy for tests on reconstituted Pisa Clay are shown in Figure 8. Simulations by the AI3-SKH model, which are very similar to the simulations by the 3-SKH model, are also included. It is clear that kinematic hardening models can predict the rotated shape of the contours of the equal specific strain energy (and therefore rotated "yield" surface defined by 'bilinear' method) without introducing rotation of the state boundary surface.

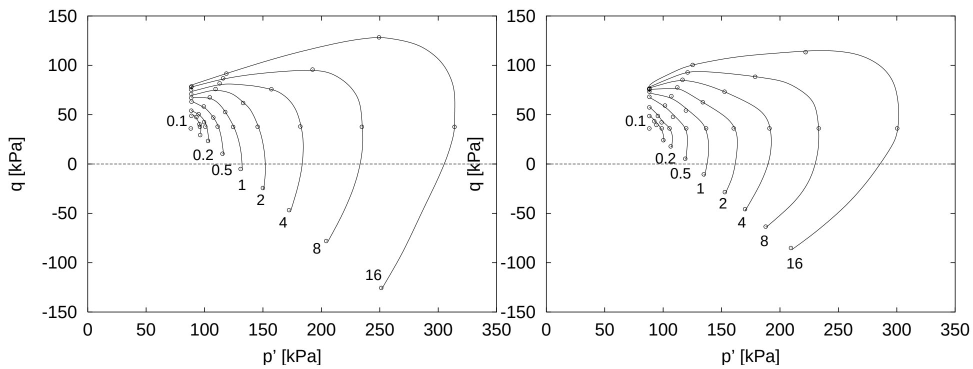

Figure 8. *Contours of the equal specific strain energy (in kJ/m^3), experimental data (after after Callisto, 1996) (left) and simulations by the AI3-SKH model (right)*

5. Conclusions

A simple extension of an existing kinematic hardening constitutive model (the 3-SKH model) using a non–associated flow rule is proposed in the paper. The modified constitutive model does not require any additional model parameters. The model predicts a smaller ratio of volumetric to shear strain increment in triaxial compression than the original model and it does not change this ratio in triaxial extension. This is in accordance with experimental data. Both original and modified models are capable of predicting the apparently rotated shape of the "state boundary surface", when it is defined by bilinear method and preserve the experimentally confirmed non–rotated shape when defined by normalization with respect to specific volume.

The proposed approach to numerical modelling of anistropically consolidated clays is generally applicable also to other existing kinematic hardening models. It allows a simpler formulation for numerical models than the widely used approach based on rotation of the state boundary surface in the direction of the K_0 line.

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