COMPARISON OF ELASTO-PLASTIC AND HYPOPLASTIC MODELLING OF STRUCTURED CLAYS

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Abstract: The paper presents an elasto-plastic equivalent of an existing hypoplastic model for structured clays. Both models are characterised by the same number of parameters with a similar physical interpretation. It is demonstrated that due to the non-linear nature of the hypoplastic equation the hypoplastic model leads to more accurate predictions than its simple elasto-plastic equivalent. The model predicts, in agreement with experiment, smooth structure-degradation process, which takes place also inside the state boundary surface.

INTRODUCTION

Although the recent constitutive models for structured clays may be based on fundamentally different mathematical backgrounds, their conceptual structure is often very similar. They are usually based on the existing models for reconstituted soils in which the size (and in some cases also the shape) of the *state boundary surface* (SBS) is modified.

The aim of the paper is a further evaluation of the hypoplastic model for structured clays by Mašín (2006a). Predictions by the model are compared with its elasto-plastic alternative developed in the paper. The Structured modified Cam clay (SMCC) model has a similar structure degradation law and the same number of parameters with an equivalent physical meaning as the hypoplastic model. Thus both the models are characterised by the same calibration procedure and the same complexity from the standpoint of a practising engineer.

All simulations with the hypoplastic model are taken over from Mašín (2006a). The aim of the present work is to supplement these simulations by predictions of a simple elasto-plastic model and thus to reveal merits of the non-linear character of the hypoplastic formulation.

CONSTITUTIVE MODELS

Hypoplastic model for clays with meta-stable structure

A hypoplastic model for clays with meta-stable structure (Mašín 2006a) has been developed by introducing a structure degradation law into the hypoplastic model for clays by Mašín (2005). Incorporation of meta-stable structure into hypoplasticity has been discussed elsewhere (Mašín 2006a; Mašín 2006b). The model assumes additional state variable *sensitivity* s^h, defined as the ratio of the sizes of SBS of structured and reference materials. Sensitivity is in the case of the hypoplastic model measured along a constant volume section through SBS, see Fig. 1a. The rate formulation of sensitivity reads

$$\dot{s}^h = -\frac{k}{\lambda^*}(s^h - s^h_f)\dot{\epsilon}^d \tag{1}$$

where k, s^h_f (final sensitivity) and λ^* are parameters and $\dot{\epsilon}^d$ is the damage strain rate, defined as

$$\dot{\epsilon}^d = \sqrt{(\dot{\epsilon}_v)^2 + \frac{A}{1-A}(\dot{\epsilon}_s)^2} \tag{2}$$

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$\dot{\epsilon}_v$ and $\dot{\epsilon}_s$ denote volumetric and shear strain rates respectively and A is a model parameter. For further details of the mathematical structure of the model the reader is referred to Mašín (2006a).

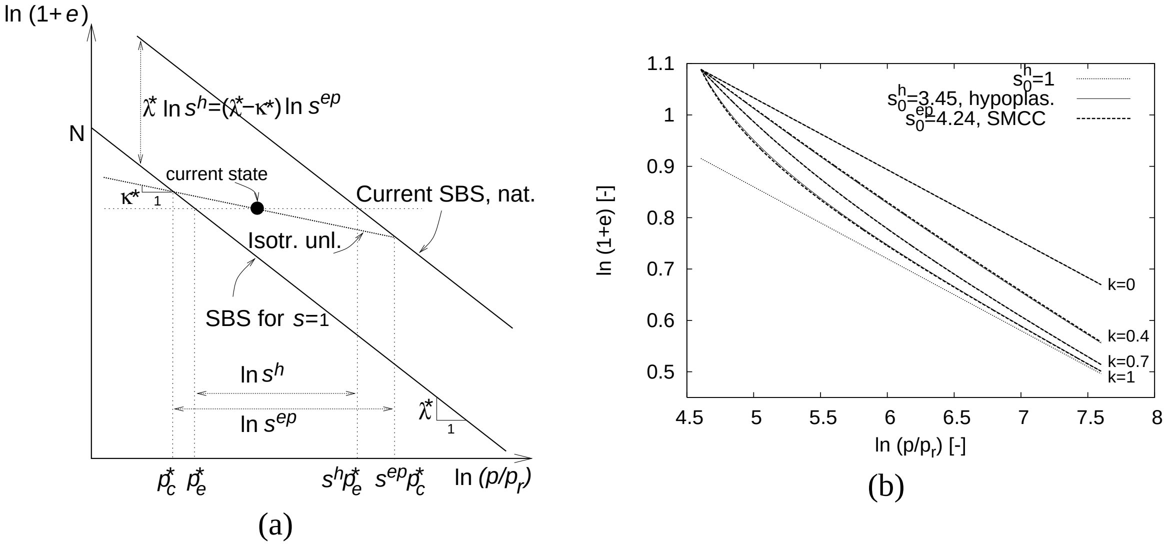


Figure 1. (a) Definitions of sensitivities s^{ep} and s^h, quantities p_c^* and p_e^* and material parameters N, λ^* and κ^*. (b) Demonstration of similarity of the two structure degradation laws on the basis of an isotropic compression test. p_r is a reference stress 1 kPa.

Structured modified Cam clay model

The SMCC model, based on the Modified Cam clay model by Roscoe and Burland (1968) enhanced by Butterfield's (1979) compression law, has been developed as an elasto-plastic equivalent of the hypoplastic model by Mašín (2006a). The mathematical formulation of the model is similar to other elasto-plastic models for structured soils, such as the models by Liu and Carter (2002) or Baudet and Stallebrass (2004).

As commonly in elasto-plastic models, sensitivity s^{ep} is measured along the *elastic wall*, not along the constant volume section through SBS as in hypoplasticity (see Fig. 1a). s^{ep} thus represents the ratio of the sizes of yield surfaces of natural and reference materials. From Fig. 1a it is clear that

$$s^{ep} = \left(s^h\right)^{\left(\frac{\lambda^*}{\lambda^*-\kappa^*}\right)} \tag{3}$$

The rate formulation for sensitivity s^{ep} reads

$$\dot{s}^{ep} = -\frac{k}{\lambda^* - \kappa^*}(s^{ep} - s_f^{ep})\dot{\epsilon}^d \tag{4}$$

and the damage strain rate is defined as

$$\dot{\epsilon}^d = \sqrt{(\dot{\epsilon}_v^p)^2 + \frac{A}{1-A}(\dot{\epsilon}_s^p)^2} \tag{5}$$

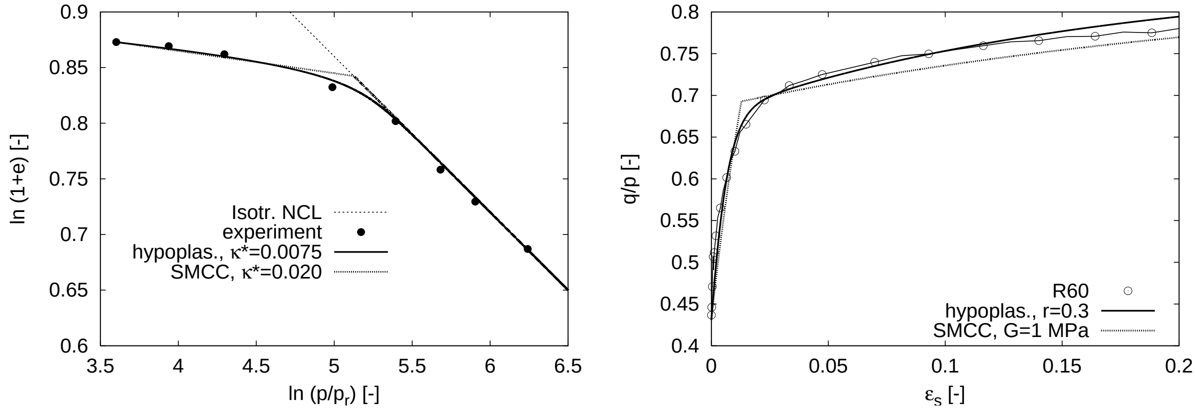
where $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_s^p$ denote *plastic* volumetric and shear strain rates respectively. A complete mathematical formulation of the SMCC model is given in Appendix.

From Eqs. (1,2) and (4,5) it is clear that the structure degradation laws of hypoplastic and SMCC models are not exactly equivalent. In order to compare both formulations, simulations of isotropic compression of isotropically normally consolidated specimens with varying parameter

k are plotted in Fig. 1b. The figure demonstrates that for the same values of the parameter k both laws yield similar rates of structure degradation and thus a direct comparison of hypoplastic and SMCC models is possible.

EVALUATION OF THE MODELS

The two constitutive models have been evaluated on the basis of laboratory experiments on natural and reconstituted Pisa clay (Callisto 1996; Callisto and Calabresi 1998) and natural Bothkennar clay (Smith et al. 1992). The details of analyses and predictions by the hypoplastic model are presented in Mašín (2006a).



(a) (b)

Figure 2. (a) Calibration of the parameters N, λ^* and κ^* of hypoplastic and SMCC models (isotropic compression test on reconstituted Pisa clay from Callisto 1996); (b) Calibration of the parameter r of the hypoplastic model and G of the SMCC model (data from Callisto and Calabresi 1998).

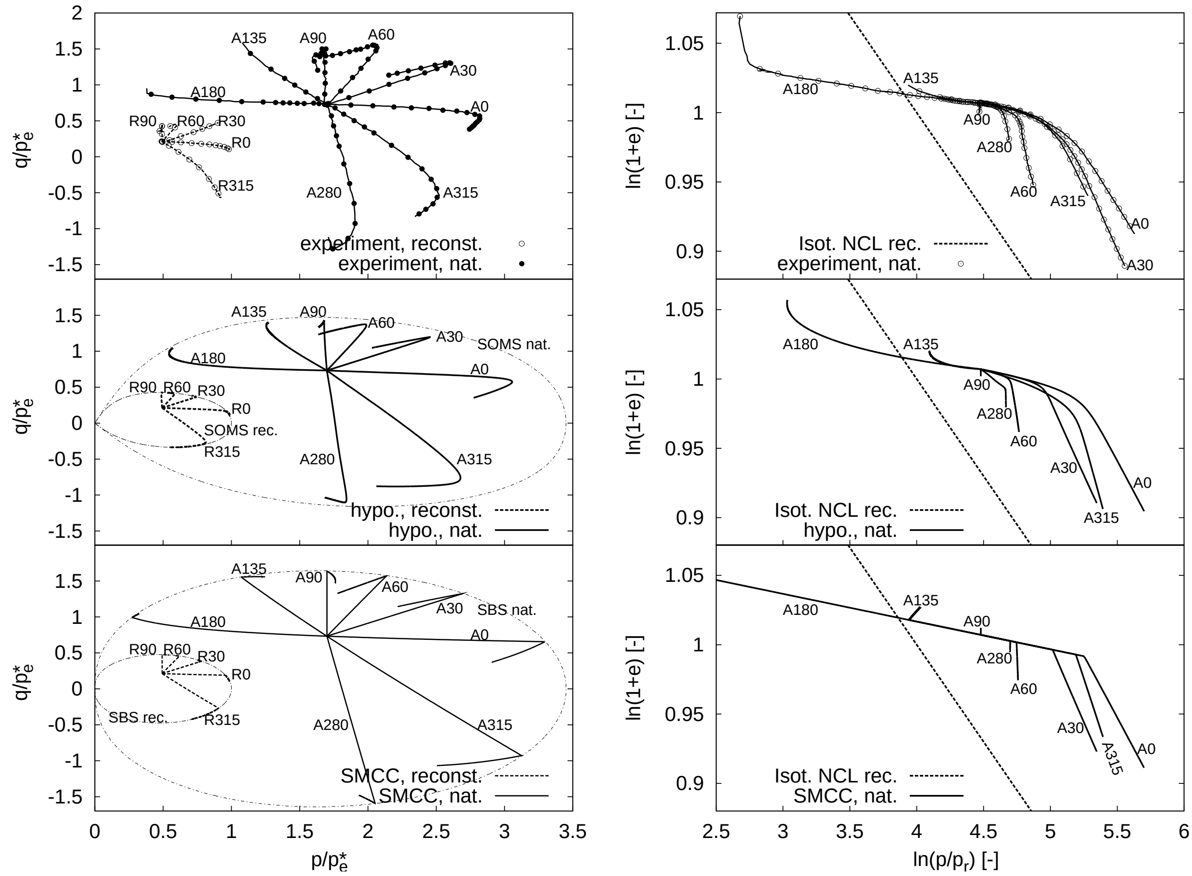
In the case of Pisa clay, all parameters with the exception of parameters that control the influence of structure (φ_c/r, λ^*, κ^*, N and r/G) were found by simulating experiments on *reconstituted* Pisa clay. Fig. 2 demonstrates the calibration of the parameters N, λ^* and κ^* and the parameters that control the shear stiffness, i.e. G (SMCC) and r (hypoplasticity). The structure-related parameters k, A, and s_f^h/s_f^{ep} were found by direct evaluation of experimental data on natural Pisa clay. In the case of Bothkennar clay, experiments on reconstituted soil, which would be equivalent to the simulated experiments on natural clay were not available, thus all parameters were found by simulation of experiments on natural clay. The parameters of both the models are summarised in Tab. 1.

Table 1. Parameters of the hypoplastic and SMCC models for Pisa and Bothkennar clays.

hypoplasticity	φ_c	λ^*	κ^*	N	r	k	A	s_f^h
Pisa	$21.9°$	0.14	0.0075	1.56	0.3	0.4	0.1	1
Bothkennar	$35°$	0.119	0.003	1.344	0.07	0.35	0.5	1
SMCC	M	λ^*	κ^*	N	G	k	A	s_f^{ep}
Pisa	0.85	0.14	0.02	1.56	1 MPa	0.4	0.1	1
Bothkennar	1.42	0.119	0.01	1.344	2 MPa	0.35	0.5	1

Fig. 3 shows the results of the simulations of experiments on Pisa clay, namely stress paths normalised by the Hvorslev equivalent pressure p_e^* (a) and the response in $\ln(p/p_r)$ vs.

$\ln(1+e)$ space (b). The normalised stress paths of natural Bothkennar clay are in Fig. 4a, ϵ_s vs. q curves in Fig. 4b. Figures 3 and 4 demonstrate some common features and some differences in predictions by the hypoplastic and SMCC models. Both models predict an apparently similar shape of the SBS and, in general, a similar stress-strain behaviour at larger strains. The main difference stems from the non-linear character of the hypoplastic equation that facilitates the non-linear response also inside the SBS, with a gradual decrease of shear and bulk moduli and a smooth structure-degradation process.

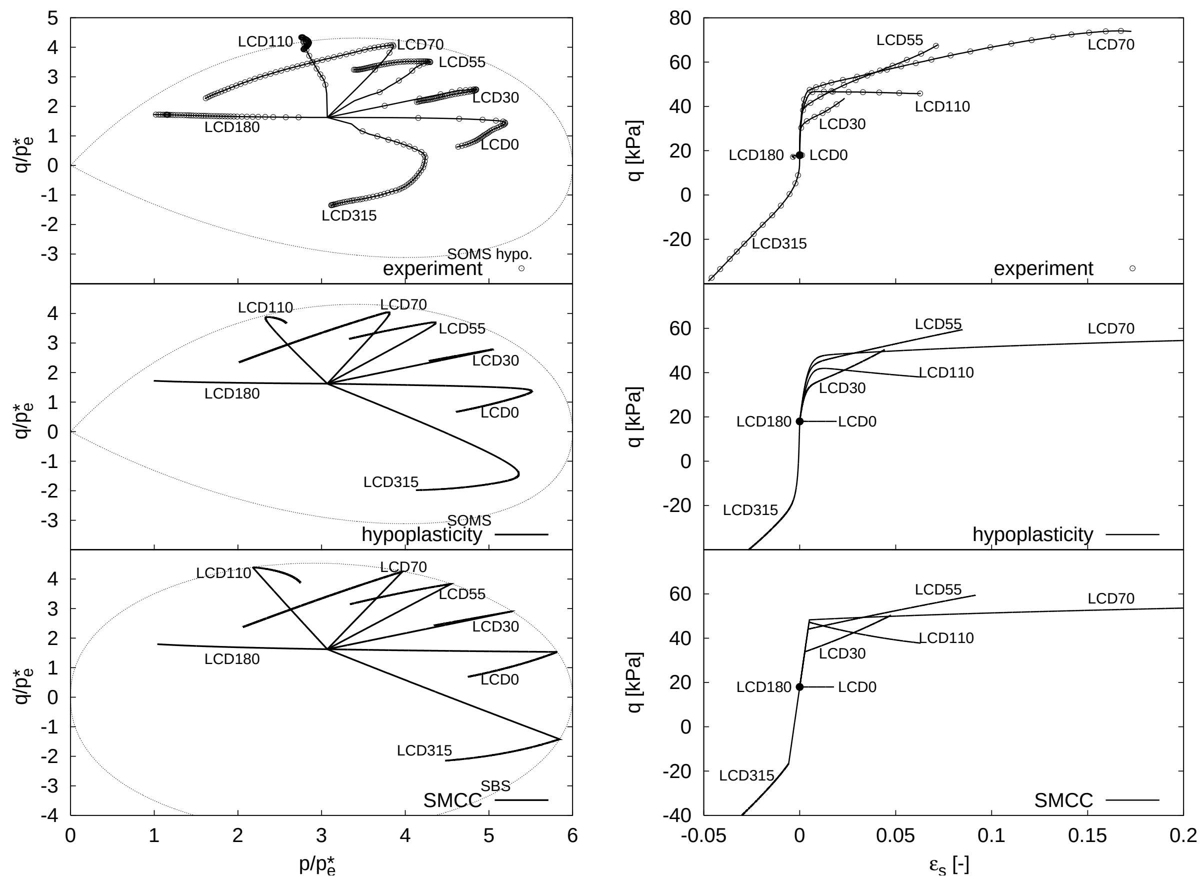


(a) (b)

Figure 3. (a) normalised stress paths of the natural and reconstituted Pisa clay and (b) experiments on natural Pisa clay plotted in the $\ln(p/p_r)$ vs. $\ln(1+e)$ space. Experimental data and predictions by the hypoplastic and SMCC models.

CONCLUDING REMARKS

The presented simulations demonstrate the well-known shortcoming of the SMCC model, its elastic behaviour inside the SBS. Many advanced elasto-plastic constitutive models overcome this problem, for example by introducing a kinematic hardening yield surface (among others see Baudet and Stallebrass 2004). These enhancements, however, often significantly increase the complexity of the mathematical formulation of the models and increase the number of parameters, which is a limiting factor for the applicability of the models for practical engineering purposes. The paper aimed to demonstrate that hypoplasticity, which requires only a limited number of material parameters (equivalent to the most simple elasto-plastic models, such as the SMCC model) is a good alternative to advanced elasto-plastic models for structured clays.



(a) (b)

Figure 4. (a) normalised stress paths and (b) ϵ_s vs. q curves from experiments on natural Bothkennar clay. Experimental data and predictions by the hypoplastic and SMCC models.

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APPENDIX

The appendix presents a complete mathematical formulation of the Structured Modified Cam clay (SMCC) model. The rate formulation of the model reads

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{\mathcal{D}}^e : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^p) \tag{6}$$

The elastic stiffness matrix $\boldsymbol{\mathcal{D}}^e$ is calculated from the shear modulus G (parameter) and bulk modulus K, related to the parameter κ^* via $K = p/\kappa^*$, by

$$\boldsymbol{\mathcal{D}}^e = \left(K - \frac{2}{3}G\right) \mathbf{1} \otimes \mathbf{1} + 2G\boldsymbol{\mathcal{I}} \tag{7}$$

Yield surface (f) is associated with the plastic potential (g) surface

$$f = g = q^2 + M^2 p \left(p - s^{ep} p_c^*\right) \tag{8}$$

M is the model parameter, s^{ep} (sensitivity) is the state variable and the quantity p_c^* is related to the state variable e (void ratio) through the equation

$$p_c^* = p_r \exp\left(\frac{N - \kappa^* \ln\left(p/p_r\right) - \ln\left(1+e\right)}{\lambda^* - \kappa^*}\right) \tag{9}$$

where p_r is the reference stress 1 kPa and N and λ^* are model parameters. Inside the yield surface ($f < 0$), $\dot{\boldsymbol{\epsilon}}^p = \mathbf{0}$. For stress states on the yield surface, the plastic strain rate is given by:

$$\dot{\boldsymbol{\epsilon}}^p = \frac{\langle \mathbf{m} : \boldsymbol{\mathcal{D}}^e : \dot{\boldsymbol{\epsilon}} \rangle}{H + \mathbf{m} : \boldsymbol{\mathcal{D}}^e : \mathbf{m}} \, \mathbf{m} \tag{10}$$

where the operator $\langle x \rangle := (x + |x|)/2$ denotes the positive part of any scalar function x, H is the plastic modulus calculated from the consistency condition

$$H = \frac{M^2 p p_c^*}{\lambda^* - \kappa^*} \left[s^{ep} \operatorname{tr}(\mathbf{m}) - k \left(s^{ep} - s_f^{ep}\right) \sqrt{\operatorname{tr}^2(\mathbf{m}) + \left(\frac{A}{1-A}\right) \frac{2}{3} \operatorname{dev}(\mathbf{m}) : \operatorname{dev}(\mathbf{m})} \right] \tag{11}$$

and the tensor $\mathbf{m}$ is calculated by:

$$\mathbf{m} = \frac{\partial f}{\partial \boldsymbol{\sigma}} = \frac{M^2(2p - s^{ep} p_c^*)}{3} \mathbf{1} + 3 \operatorname{dev}(\boldsymbol{\sigma}) \tag{12}$$

s_f^{ep} and k are model parameters. Evolution of state variables is governed by equations:

$$\dot{e} = -\left(1+e\right) \dot{\epsilon}_v \qquad \dot{s}^{ep} = -\frac{k}{\lambda^* - \kappa^*} (s^{ep} - s_f^{ep}) \sqrt{\left(\dot{\epsilon}_v^p\right)^2 + \frac{A}{1-A} \left(\dot{\epsilon}_s^p\right)^2} \tag{13}$$

$\dot{\epsilon}_v^p$ and $\dot{\epsilon}_s^p$ are rates of plastic volumetric and shear strains respectively and A is model parameter.