A hypoplastic constitutive model for clays with meta-stable structure

David Mašín

Charles University
Institute of Hydrogeology, Engineering Geology and Applied Geophysics
Albertov 6
12843 Prague 2, Czech Republic
E-mail: masin@natur.cuni.cz
Tel: +420-2-2195 1552, Fax: +420-2-2195 1556

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Abstract

A new hypoplastic model for clays with meta-stable structure is presented in the paper. A new method for incorporation of structure effects into hypoplastic models based on the modification of *barotropy* and *pyknotropy* factors is proposed and applied to an existing hypoplastic model for reconstituted clays. The new model is characterised by a simple calibration procedure and a small number of parameters. This makes the model particularly suitable for practical applications. The model is evaluated using experimental data on two natural soft clays. Thanks to the incrementally non-linear character of the hypoplastic equation, the proposed model predicts behaviour of overconsolidated clays comparably to advanced kinematic hardening elasto-plastic models.

Keywords: Constitutive relations; hypoplasticity; clays; structure of soils

Introduction

Constitutive modelling of natural structured clays has observed a notable development in past years. The research is driven by need for suitable design procedures, which would allow a practising engineer to perform analyses which are reliable and safe, but still sufficiently cheap. Apart from the accuracy in reproducing the soil behaviour, the model used for this purpose should be easy to calibrate on the basis of laboratory experiments performed in a standard experimental equipment available in practice. The main objective of the research presented in this paper is to provide an advanced constitutive model for structured clays that fulfills these requirements.

Most of the currently available constitutive models, which describe the destructuration processes in natural clays are developed within the framework of elasto-plasticity and visco-plasticity and may be seen as different extensions of the classical critical state model, developed at Cambridge University in 1960's (Roscoe and Burland 1968). This model is usually modified by incorporating the second hardening law, which describes the progressive changes of structure of natural clay. The most simple models (such as the model by Liu and Carter 2002) do not assume any other alteration of the original model, Wheeler et al. (2003) include anisotropic effects by modifying the shape of the state boundary surface. These models, however, are not capable of predicting non-linearity of behaviour of overconsolidated soils. This shortcoming is overcome by more advanced models that make use of the kinematic hardening plasticity (Mróz et al. 1979), such as models by Baudet and Stallebrass (2004), Rouainia and Muir Wood (2000), Kavvadas and Amorosi (2000) and

Gajo and Muir Wood 2001. Different approaches to treat the behaviour of structured soils include the multilaminate framework (Cudny and Vermeer 2004), visco-plasticity (Rocchi et al. 2003), and super/subloading yield surface approach (Asaoka 2005). A common feature of these models is that the improvement in accuracy of predictions is often paid by an increase of complexity of calibration procedures and mathematical formulation, thus reducing their suitability for application in a routine design.

In the text, the usual sign convention of soil mechanics (compression positive) is adopted throughout. In line with the Terzaghi principle of effective stress, all stresses are *effective* stresses. Common tensor notation (see, e.g., Mašín and Herle 2005) is used.

Reference constitutive model

The theory of hypoplasticity (Kolymbas 1978, Kolymbas 1991), developed independently at Universities of Karlsruhe and Grenoble (see Tamagnini et al. 2000), was in the past applied successfully in the development of constitutive relations for granular materials (Gudehus 1996, von Wolffersdorff 1996, Chambon et al. 1994). More recently, the research focused on the development of hypoplastic constitutive models for fine-grained soils (Niemunis 1996, Gudehus 2004, Herle and Kolymbas 2004). Mašín (2005) proposed a new hypoplastic constitutive model for clays, which combines mathematical structure of hypoplastic models with the basic principles of the critical state soil mechanics and the Modified Cam clay model. Predictive capabilities of this model, compared with other advanced constitutive models, have been demonstrated, e.g., by Mašín et al. (2006) and Hájek and Mašín (2006). The hypoplastic model for clays will be used as a reference model for the proposed modification.

The rate formulation of hypoplastic models is, in general (Lanier et al. 2004), characterised by a single equation (Gudehus 1996)[1]

$$\dot{\boldsymbol{\sigma}} = f_s \boldsymbol{\mathcal{L}} : \dot{\boldsymbol{\epsilon}} + f_s f_d \mathbf{N} \left\| \dot{\boldsymbol{\epsilon}} \right\| \tag{1}$$

where $\boldsymbol{\mathcal{L}}$ and $\mathbf{N}$ are fourth- and second-order constitutive tensors respectively, f_s is *barotropy* factor that incorporates the influence of the mean stress and f_d is a *pyknotropy* factor, which controls

[1] To be more precise, the rate formulation of hypoplastic models reads $\mathring{\boldsymbol{\sigma}} = f_s \boldsymbol{\mathcal{L}} : \mathbf{D} + f_s f_d \mathbf{N} \left\| \mathbf{D} \right\|$, where $\mathring{\boldsymbol{\sigma}}$ is the objective stress rate and $\mathbf{D}$ the Euler's stretching tensor. See, e.g., Kolymbas and Herle (2003) and Beghini and Bažant (2004) for details.

the influence of the relative density (overconsolidation ratio). Cauchy stress $\boldsymbol{\sigma}$ and void ratio e are considered as state variables. Complete mathematical formulation of the reference model is given in Appendix.

The model requires five material parameters, namely φ_c, N, λ^*, κ^* and r. φ_c is the critical state friction angle. Parameters N and λ^* define the position and shape of the isotropic virgin compression line with the formulation according to Butterfield (1979):

$$\ln(1+e) = N - \lambda^* \ln\left(\frac{p}{p_r}\right) \tag{2}$$

where p_r is the reference stress 1kPa. The parameter κ^* determines bulk modulus at overconsolidated states and the parameter r controls shear modulus. Although direct calibration of parameters κ^* and r is possible (Mašín 2005), it is suggested to determine their values by means of parametric studies.

Conceptual approach for incorporation of structure effects into constitutive models

A conceptual framework for the behaviour of structured fine-grained soils was presented by Cotecchia and Chandler (2000). They demonstrated that the influence of structure in fine-grained soils can be quantified by the different sizes of the state boundary surfaces[2] of the structured and reference materials (Fig. 1), where as the reference material is usually considered soil reconstituted under standard conditions (Burland 1990). Cotecchia and Chandler (2000) show that assuming a geometric similarity between the state boundary surfaces of natural and reference materials appears to be a reasonable approximation, although strongly anisotropic natural soils may exhibit SBS which is not symmetric about the isotropic axis.

These observations are, in principle, applied in most of the currently available constitutive models for structured soils. In general, at least one additional state variable describing the effects of structure is needed, namely the ratio of sizes of SBSs of natural and reference materials, referred to as 'sensitivity' (s). s represents natural fabric and degree of bonding between soil particles. The limit value usually characterise the reference soil ($s = 1$), although higher values may be reasonable for soils with 'stable' elements of structure caused by natural fabric (Baudet and Stallebrass 2004).

[2]State boundary surface (SBS) is defined as a boundary of all possible states of a soil element in the stress-void ratio space.

s is usually considered as a function of accumulated plastic strain.

Incorporation of structure effects into hypoplasticity

As opposed to the most elasto-plastic models, the mathematical formulation of hypoplastic models does not include explicitly the state boundary surface. However, Mašín and Herle (2005) demonstrated that the model formulation allows us to derive an expression for the so-called swept-out-memory (SOM) surface, which is a close approximation of the state boundary surface.

They have shown that for any permitted stress state it is possible to calculate explicitly the value of the *pyknotropy* factor f_d on the swept-out-memory surface:

$$f_d = \| f_s \boldsymbol{\mathcal{A}}^{-1} : \mathbf{N} \|^{-1} \tag{3}$$

where the fourth-order tensor $\boldsymbol{\mathcal{A}}$ is expressed as

$$\boldsymbol{\mathcal{A}} = f_s \boldsymbol{\mathcal{L}} - \frac{1}{\lambda^*} \boldsymbol{\sigma} \otimes \mathbf{1} \tag{4}$$

Equations (3) and (31) can be combined to find the expression for the Hvorslev equivalent pressure p_e^* for a given stress state $\boldsymbol{\sigma}$ on the swept-out-memory surface and thus to determine the shape of the SOM surface in the normalised space $\boldsymbol{\sigma}/p_e^*$:

$$p_e^* = 2p \| f_s \boldsymbol{\mathcal{A}}^{-1} : \mathbf{N} \|^{1/\alpha} \tag{5}$$

where the Hvorslev equivalent pressure p_e^* on the isotropic normal compression line is defined as (from (2), Fig. 3)

$$p_e^* = p_r \exp \left[\frac{N - \ln(1+e)}{\lambda^*} \right] \tag{6}$$

Since the swept-out-memory surface is not given *a priori*, its shape is dependent on the model parameters, namely φ_c and the ratio κ^*/λ^*. For parameters typical to fine-grained soils its shape is similar to the state boundary surface of the Modified Cam clay model (see Fig. 2 for different parameter sets).

As summarised in the Introduction, constitutive modelling of structured soils using the framework of elasto-plasticity has recently undergone a notable development. Only few attempts, however, have been made to incorporate structure effects into hypoplastic models. Bauer and Wu (1993) and Bauer and Wu (1994) enhanced the early version of the hypoplastic model for granular materials (Wu 1992), which considers Cauchy stress $\boldsymbol{\sigma}$ the only state variable, by the so-called structure tensor $\mathbf{S}$. The Cauchy stress $\boldsymbol{\sigma}$ is in the model replaced by the "transformed stress tensor" $\boldsymbol{\sigma}^*$, defined as

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma} + \mathbf{S} \tag{7}$$

This transformation shifts the limit state locus in the stress space, thus enabling modelling the cohesive behaviour of cemented materials. A suitable evolution equation for the structure tensor $\mathbf{S}$ then allows us to simulate degradation of cementation bonds.

A different approach for incorporating the structure effects into hypoplastic model is proposed in the present work. Soil with stable structure (constant sensitivity) is considered first, following Ingram (2000). As in the present work sensitivity s is measured along *constant volume* sections through the state boundary surfaces (Fig. 3), the Hvorslev equivalent pressure of the structured material is calculated by sp_e^* (Fig. 3). It follows from the expression of the SOM surface (Eqs. (3)-(6)) that the reference hypoplastic model may be modified for clays with stable structure by a simple replacement of p_e^* in the expression for f_d (see Eq. (31) in Appendix) by sp_e^*:

$$f_d = \left(\frac{2p}{sp_e^*}\right)^{\alpha} \tag{8}$$

Eq. (8) causes that the SBS of a natural soil is s times larger than the SBS of a corresponding reference material. It also follows from Fig. 3 that the normal compression line of a natural soil is shifted along $\ln(1+e)$ axis in the $\ln(1+e)$ vs. $\ln p$ space by $\lambda^* \ln s$. Additional enhancement by the transformed stress tensor $\boldsymbol{\sigma}^*$ (7) would shift the SBS along the isotropic axis and thus would allow us to model true cohesive behaviour due to cementation bonds (Mašín 2006). For simplicity, the latter modification is omitted in this Note.

Second, the model is modified to predict the structure degradation. The proposed evolution equation for sensitivity s reads (similarly to Baudet and Stallebrass 2004)

$$\dot{s} = -\frac{k}{\lambda^*}(s - s_f)\dot{\epsilon}^d \tag{9}$$

where k is a constitutive parameter that controls the rate of the structure degradation and s_f is the final sensitivity. The damage strain $\dot{\epsilon}^d$ is defined by

$$\dot{\epsilon}^d = \sqrt{(\dot{\epsilon}_v)^2 + \frac{A}{1-A}(\dot{\epsilon}_s)^2} \tag{10}$$

with the parameter A, which controls the relative importance of the volumetric and shear components (similarly to Rouainia and Muir Wood 2000). Obviously, Eq. (10) does not allow modelling purely deviatoric structure degradation process ($A \to 1$). Nevertheless, the research by Rouainia and Muir Wood (2000), Gajo and Muir Wood (2001) and Callisto and Rampello (2004) indicate that the value of the parameter A may be for most clays expected in the range ($0 < A < 0.5$).

In order to incorporate the variable sensitivity into the hypoplastic model, the *barotropy* factor f_s needs to be modified to ensure consistency between the model predictions and the structure degradation law (9). Formulation of the model for the isotropic compression from the isotropic normally compressed state is given by

$$\dot{p} = -\left[\frac{1}{3(1+e)} f_s \left(3 + a^2 - 2^\alpha a\sqrt{3}\right)\right]\dot{e} \tag{11}$$

The isotropic normal compression line of the model incorporating structure reads (see Fig. 3)

$$\ln(1+e) = N + \lambda^* \ln s - \lambda^* \ln\left(\frac{p}{p_r}\right) \tag{12}$$

Time differentiation of (12) results in

$$\frac{\dot{e}}{1+e} = \lambda^* \left(\frac{\dot{s}}{s} - \frac{\dot{p}}{p}\right) \tag{13}$$

The isotropic formulation of the structure degradation law (9-10) is

$$\dot{s} = \frac{k}{\lambda^*}(s - s_f)\frac{\dot{e}}{1+e} \tag{14}$$

Combination of (13) and (14) yields

$$\frac{\dot{p}}{p} = -\left[\frac{s - k(s-s_f)}{\lambda^* s}\right]\frac{\dot{e}}{1+e} \tag{15}$$

which may be compared with (11) to find an expression for the *barotropy* factor f_s of the new hypoplastic model:

$$f_s = S_i \frac{3p}{\lambda^*}\left(3 + a^2 - 2^\alpha a\sqrt{3}\right)^{-1} \tag{16}$$

with the factor

$$S_i = \frac{s - k(s - s_f)}{s} \tag{17}$$

Thus the factor f_s of the modified model reads $f_s = S_i f_{sr}$, where f_{sr} is the barotropy factor of the reference model (see Eq. (30) in Appendix).

It follows from (1) that the factor f_s controls the directional tangential stiffness of material (in terms of response envelopes (Gudehus 1979) it controls their size). Therefore, the decrease of the stiffness in isotropic compression to ensure consistency with the structure degradation law (Eqs. (11)-(17)) has an undesired effect that also shear stiffness (controlled by parameter r) and stiffness in isotropic unloading (controlled by parameter κ^*) is decreased, see Fig. 4 (case A). Manipulation with the model reveals that the physical meaning of the parameters r and κ^* is retained if they are both scaled by the factor S_i (Fig. 4, case B). Therefore, modification of scalar factors c_1 (23) and α (24) is required. They now read

$$c_1 = \frac{2\left(3 + a^2 - 2^\alpha a\sqrt{3}\right)}{9rS_i} \qquad\qquad \alpha = \frac{1}{\ln 2}\ln\left[\frac{\lambda^* - \kappa^* S_i}{\lambda^* + \kappa^* S_i}\left(\frac{3 + a^2}{a\sqrt{3}}\right)\right] \tag{18}$$

Equations (8–10), (16–17) and (18) are the only modifications of the reference hypoplastic model. The new model assumes one additional state variable s and three additional parameters: k, A, and s_f. Their calibration procedure is detailed in the following text.

Model performance and calibration

The performance of the proposed hypoplastic model will be evaluated using the concept of the *normalised incremental stress response envelopes* (NIREs, see Fig. 5). They have been introduced in Mašín and Herle (2005) and follow directly from the concept of incremental response envelopes (Tamagnini et al. 2000) and rate response envelopes (Gudehus 1979).

Figures 6a and 6b show the NIREs for different $R_{\Delta\boldsymbol{\epsilon}} = \|\Delta\boldsymbol{\epsilon}\|$ for natural and reconstituted specimens of Pisa clay (see the next section), with symbols for isotropic and constant volume loading and unloading. For small $R_{\Delta\boldsymbol{\epsilon}}$ (states well inside the swept-out-memory surface, Fig. 6a) the NIREs of the natural and reconstituted clays are similar in shape, the sizes of the NIREs of the natural clay are s_0 times larger than corresponding NIREs of the reconstituted clay (where s_0 is the initial sensitivity of the natural clay). We see that although the damage strain (10) is defined in terms of total strain rates (instead of plastic strain rates as usual in elasto-plastic models), the model predicts only minor structure degradation for states inside the SBS. Minor structure degradation also inside the SBS is supported by Takahashi et al. (2005).

For larger $R_{\Delta\boldsymbol{\epsilon}}$ (Fig. 6b) the NIREs of the reconstituted clay coincide with its swept-out-memory surface. The progressive structure degradation of the natural clay, however, causes the NIREs of the natural clay to shrink towards the swept-out-memory surface of the reconstituted material. The heart-like shape of the NIREs of the natural clay for $R_{\Delta\boldsymbol{\epsilon}} > 8\%$ is caused by the low value of the parameter A (Tab. 1), which causes more significant influence of the volumetric strain component on the structure degradation.

The influence of the parameter k on model predictions is demonstrated in Fig. 7a. The value of the parameter k was varied, while other model parameters (Tab. 1) were kept unchanged. The figure demonstrates the faster structure degradation for larger values of the parameter k. The influence of the parameter A is shown in Fig. 7b. Larger value of the parameter A increases the influence of shear strains on the structure degradation and thus flattens the NIREs. The common point of all NIREs is at the isotropic stress state. Therefore, the parameter k may be calibrated independently of the parameter A on the basis of an isotropic compression test on natural soil. The parameter A is calibrated with the already known value of k using an experiment where significant shear strains develop.

The initial value of sensitivity may be determined from an assumption of a geometric similarity between SBSs of natural and reconstituted soil (Cotecchia and Chandler 2000) as a ratio of undrained shear strengths of natural and reconstituted soil, or as a ratio of stresses at gross yield in compression tests (e.g., K_0 or isotropic) on natural specimens and equivalent stresses at corresponding normal compression lines of a reconstituted soil. The final value of sensitivity s_f may be derived from compression tests on natural and reference materials performed to very large strains (Baudet 2001).

Evaluation of model predictions

The proposed model will be evaluated on the basis of laboratory experiments on two natural clays with meta-stable structure.

Callisto and Calabresi (1998) reported laboratory experiments on natural Pisa clay. Drained probing tests were performed, with rectilinear stress paths having different orientations in the stress space. In addition to the tests on natural Pisa clay, experiments with the same stress paths were performed on reconstituted clay. Tests are labelled by prefix 'A' and 'R' for natural and reconstituted clay respectively, followed by the angle of stress paths in the $q:p$ space (measured in degrees anti-clockwise from the isotropic loading direction).

All the parameters of the proposed hypoplastic model, with the exception of parameters related to the effects of structure (k, A and s_f), were be calibrated solely using laboratory experiments on the reconstituted Pisa clay. The parameters N, λ^* and κ^* were calibrated on the basis of an isotropic compression test on reconstituted Pisa clay (Callisto 1996, Fig. 8a). Critical state friction angle φ_c has been found by evaluating the data from shear tests, parameter r has been calibrated on the basis of a parametric study using a single shear test (R60, Fig. 8b).

For the calibration of the structure-related parameters k, A and s_f, it has been *assumed*, following Callisto and Rampello (2004), that the experimental procedures adopted for preparation of reconstituted specimens reproduced correctly the stress history of the Pisa clay deposit. Therefore, the stress paths of the equivalent experiments on natural and reconstituted clays should coincide, when plotted in the space normalised with respect to volume and structure $\boldsymbol{\sigma}/(p_e^* s)$ (Cotecchia and Chandler 2000). The current value of sensitivity s may be found by the time-integration of the structure degradation law (9-10):

$$s = s_f + (s_0 - s_f) \exp\left[-\frac{k}{\lambda^*}\epsilon^d\right] \tag{19}$$

where ϵ^d is the accumulated damage strain

$$\epsilon^d = \int_{t_0}^{t_1} \dot{\epsilon}^d dt \tag{20}$$

In this way, parameters k and A (and the initial value of sensitivity s) could be calibrated directly by evaluation of the experimental data, without reference to single element modelling of tests on natural clay. Calibration of the parameter k is demonstrated in Fig. 9a. The shear strains in the test A0 are negligible, thus the parameter A does not influence predictions of the structure degradation process (Fig. 7b, Eq. (10)). The value of the parameter A was calibrated with the already known value of the parameter k using results from shear tests R90 and A90 (Fig. 9b). The final value of sensitivity s_f is assumed to be equal to one. This appears to be a reasonable approximation (Baudet 2001), although no compression or shear experiment on natural clay which would lead to a full destructuration is available. Parameters of the hypoplastic model for natural Pisa clay are summarised in Table 1 and the initial values of state variables in Table 2.

The evaluated parameters were used for simulation of laboratory experiments on Pisa clay. Experimental data compared with predictions by the proposed hypoplastic model are shown in Figs. 10, 11 and 12. The figures show that the hypoplastic model, due to its non-linear nature, predicts correctly the gradual change of stiffness as the state moves towards the state boundary surface. Consequently, the model predicts in agreement with experiment smooth structure degradation process, which amplifies as the state moves towards the state boundary surface (Fig. 10b).

Performance of the model in the strain space is evaluated in Fig. 13 using the concept of incremental strain response envelopes (ISREs) (Tamagnini et al. 2000, Mašín et al. 2006), defined inversely to the incremental stress response envelopes. The hypoplastic model (Fig. 13b) predicts correctly the shape of ISREs, with softer response in compression.

Experimental database by Callisto (1996) includes undrained compression (AUC) and extension (AUE) tests on natural Pisa clay samples with the same pre-shear stress history as drained probes A0–A315. These tests were simulated with parameters evaluated using data from drained probes (Fig. 14). In compression, the proposed model predicts qualitatively correctly the shape of the stress path, but the shear stiffness and the peak friction angle are underestimated. In extension the stress-strain response is predicted correctly. However, although the final state is reproduced accurately, the model predicts significant decrease in mean stress in initial stages of the experiment that was not be observed experimentally.

Smith et al. (1992) performed a series of triaxial stress probing tests on natural Bothkennar clay. The soil, classified as a very silty clay (Hight et al. 1992), is characterised by relatively high (3-5%) organic content. The soil composition induces somewhat unusual mechanical properties with high plasticity typical to fine-grained soils combined with high critical state friction angles (Allman and Atkinson 1992). The stress-probing experiments with constant direction of stress paths in the stress space are labelled by prefix 'LCD' followed by the orientation of the stress paths in $q : p$ space.

The parameters N and λ^* for the Bothkennar clay were calibrated using results of K_0 test on a reconstituted sample (Smith et al. 1992, Fig. 18). The shape of the swept-out-memory surface of the hypoplastic model was taken into account in calculation of the parameter N from the position of the K_0 normal compression line in the $\ln(1+e) : \ln(p/p_r)$ space. The final sensitivity s_f is equal to one, as full destructuration is observed in K_0 compression experiments on natural Bothkennar clay (Smith et al. 1992, Fig. 18). Because the set of stress probing tests published by Smith et al. (1992) does not include equivalent experiments on reconstituted soil, other parameters including the initial value of sensitivity were evaluated directly using stress probing data on natural Bothkennar clay by means of parametric studies. The parameters and the initial values of state variables are summarised in Tabs. 1 and 2.

Comparison of experimental data from drained stress probing experiments on natural Bothkennar clay (Smith et al. 1992) with predictions by the proposed hypoplastic model are shown in Figs. 15–17. Similarly to predictions of tests on natural Pisa clay, the proposed model yields results which are in an agreement with experiments. The only notable difference is the normalised stress paths of the test LCD315 which, due to the shape of the swept-out-memory surface of the hypoplastic model, bends later than the experimental normalised stress path. In this case the rotated shape of the swept-out-memory surface would possibly lead to improvement of predictions. The incorporation of anisotropic effects, demonstrated within the hypoplastic framework, for example, by Wu (1998) and Niemunis (2003), is however outside the scope of this paper.

The set of parameters optimized for predictions of drained stress probing tests LCD was further used to simulate K_0 compression tests on natural Bothkennar clay. Experimental data on Laval and Sherbrooke samples from Smith et al. (1992), together with predictions by the proposed hypoplastic model, are shown in Fig. 18. It is clear that the parameters optimized for predictions of LCD tests lead to underprediction of the structure degradation process in K_0 compression, which may be possibly attributed to larger disturbance of oedometric specimens in comparison with specimens tested in triaxial apparatuses. Similar observation is reported by Callisto et al. (2002) using the kinematic hardening model for structured clays by Rouainia and Muir Wood (2000). A better fit of the experimental data is achieved by increasing the value of the parameter k ($k = 0.6$) and decreasing the initial sensitivity ($s_0 = 4$) – see also Fig. 18.

Summary and conclusions

A simple approach to incorporating structure effects into an existing hypoplastic constitutive model for reconstituted clays is presented in the paper. Unlike the previous attempts to incorporate structure effects into hypoplasticity, the proposed approach is based on the modification of the *barotropy* and *pyknotropy* factors that leads to an increase of the size of the state boundary surface predicted by the model and ensures consistency between the model predictions and the pre-defined structure degradation law. Model predictions compare well with experimental data on two natural clays. In fact, predictions of laboratory experiments on natural Pisa and Bothkennar clays presented in the paper are comparable with predictions by kinematic hardening elasto-plastic models, see Baudet (2001) and Callisto et al. (2002) for drained probing tests on natural Pisa clay and Baudet and Stallebrass (2004), Baudet (2001) and Gajo and Muir Wood (2001) for LCD tests on natural Bothkennar clay.

The proposed method for incorporating the structure effects into hypoplasticity also opens a way to model other structural effects using hypoplasticity theory, such as mechanical (Lagioia and Nova 1995) and chemical (Nova et al. 2003) debonding in cemented granular materials, simulating grain crushing (Cecconi et al. 2002), or modelling unsaturated (Alonso et al. 1990) and double-porosity materials (Mašín et al. 2005).

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Appendix

The mathematical formulation of a reference hypoplastic model for clays model is summarised briefly in the following. The rate formulation of the hypoplastic model reads

$$\dot{\boldsymbol{\sigma}} = f_s \boldsymbol{\mathcal{L}} : \dot{\boldsymbol{\epsilon}} + f_s f_d \mathbf{N} \left\| \dot{\boldsymbol{\epsilon}} \right\| \tag{21}$$

The fourth-order tensor $\boldsymbol{\mathcal{L}}$ is a hypoelastic tensor given by

$$\boldsymbol{\mathcal{L}} = 3\left(c_1\boldsymbol{\mathcal{I}} + c_2 a^2 \hat{\boldsymbol{\sigma}} \otimes \hat{\boldsymbol{\sigma}}\right) \tag{22}$$

with the two scalar factors c_1 and c_2 introduced by Herle and Kolymbas (2004) and modified by Mašín (2005):

$$c_1 = \frac{2\left(3 + a^2 - 2^\alpha a\sqrt{3}\right)}{9r} \qquad\qquad c_2 = 1 + (1 - c_1)\frac{3}{a^2} \tag{23}$$

where the scalars a and α are functions of the material parameters φ_c, λ^* and κ^*

$$a = \frac{\sqrt{3}\,(3 - \sin\varphi_c)}{2\sqrt{2}\sin\varphi_c} \qquad\qquad \alpha = \frac{1}{\ln 2}\ln\left[\frac{\lambda^* - \kappa^*}{\lambda^* + \kappa^*}\left(\frac{3 + a^2}{a\sqrt{3}}\right)\right] \tag{24}$$

The second-order tensor $\mathbf{N}$ is given by (Niemunis 2002)

$$\mathbf{N} = \boldsymbol{\mathcal{L}} : \left(Y\frac{\mathbf{m}}{\|\mathbf{m}\|}\right) \tag{25}$$

where the quantity Y determines the shape of the critical state locus in the stress space such that for $Y = 1$ it coincides with the Matsuoka and Nakai (1974) limit stress condition.

$$Y = \left(\frac{\sqrt{3}a}{3 + a^2} - 1\right)\frac{(I_1 I_2 + 9I_3)\left(1 - \sin^2\varphi_c\right)}{8I_3\sin^2\varphi_c} + \frac{\sqrt{3}a}{3 + a^2} \tag{26}$$

with the stress invariants

$$I_1 = \operatorname{tr}(\boldsymbol{\sigma}) \qquad\qquad I_2 = \frac{1}{2}\left[\boldsymbol{\sigma} : \boldsymbol{\sigma} - (I_1)^2\right] \qquad\qquad I_3 = \det(\boldsymbol{\sigma})$$

$\det(\boldsymbol{\sigma})$ is the determinant of $\boldsymbol{\sigma}$. The second-order tensor $\mathbf{m}$ has parallel in the flow rule in elasto-plasticity. It is calculated by

$$\mathbf{m} = -\frac{a}{F}\left[\hat{\boldsymbol{\sigma}} + \operatorname{dev}\hat{\boldsymbol{\sigma}} - \frac{\hat{\boldsymbol{\sigma}}}{3}\left(\frac{6\hat{\boldsymbol{\sigma}} : \hat{\boldsymbol{\sigma}} - 1}{(F/a)^2 + \hat{\boldsymbol{\sigma}} : \hat{\boldsymbol{\sigma}}}\right)\right] \tag{27}$$

with the factor F

$$F = \sqrt{\frac{1}{8}\tan^2\psi + \frac{2 - \tan^2\psi}{2 + \sqrt{2}\tan\psi\cos 3\theta}} - \frac{1}{2\sqrt{2}}\tan\psi \tag{28}$$

where

$$\tan\psi = \sqrt{3}\,\|\mathrm{dev}\,\hat{\boldsymbol{\sigma}}\| \qquad\qquad \cos 3\theta = -\sqrt{6}\frac{\mathrm{tr}\,(\mathrm{dev}\,\hat{\boldsymbol{\sigma}}\cdot\mathrm{dev}\,\hat{\boldsymbol{\sigma}}\cdot\mathrm{dev}\,\hat{\boldsymbol{\sigma}})}{[\mathrm{dev}\,\hat{\boldsymbol{\sigma}}:\mathrm{dev}\,\hat{\boldsymbol{\sigma}}]^{3/2}} \tag{29}$$

The *barotropy* factor f_s introduces the influence of the mean stress level. The way of its derivation ensures that the hypoplastic model predicts correctly the isotropic normally compressed states.

$$f_s = \frac{3p}{\lambda^*}\left(3 + a^2 - 2^\alpha a\sqrt{3}\right)^{-1} \tag{30}$$

The *pyknotropy* factor f_d incorporates the influence of the overconsolidation ratio. The critical state is characterised by $f_d = 1$ and the isotropic normally compressed state by $f_d = 2^\alpha$.

$$f_d = \left(\frac{2p}{p_e^*}\right)^\alpha \qquad\qquad p_e^* = p_r \exp\left[\frac{N - \ln(1+e)}{\lambda^*}\right] \tag{31}$$

with the reference stress $p_r = 1$ kPa. Finally, evolution of the state variable e (void ratio) is governed by

$$\dot{e} = -\,(1+e)\,\dot{\epsilon}_v \tag{32}$$

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Table 1: *Parameters of the proposed hypoplastic model for Pisa clay and Bothkennar clay.*

	φ_c	λ^*	κ^*	N	r	k	A	s_f
Pisa	21.9°	0.14	0.0075	1.56	0.3	0.4	0.1	1
Bothkennar	35°	0.119	0.003	1.344	0.07	0.35	0.5	1

Table 2: *The initial values of the state variables for natural and reconstituted Pisa clay and natural Bothkennar clay.*

p [kPa]	q [kPa]	e reconst.	e nat.	s
88.2	38	1.302	1.738	3.45
34	18	–	1.88	6

Figures

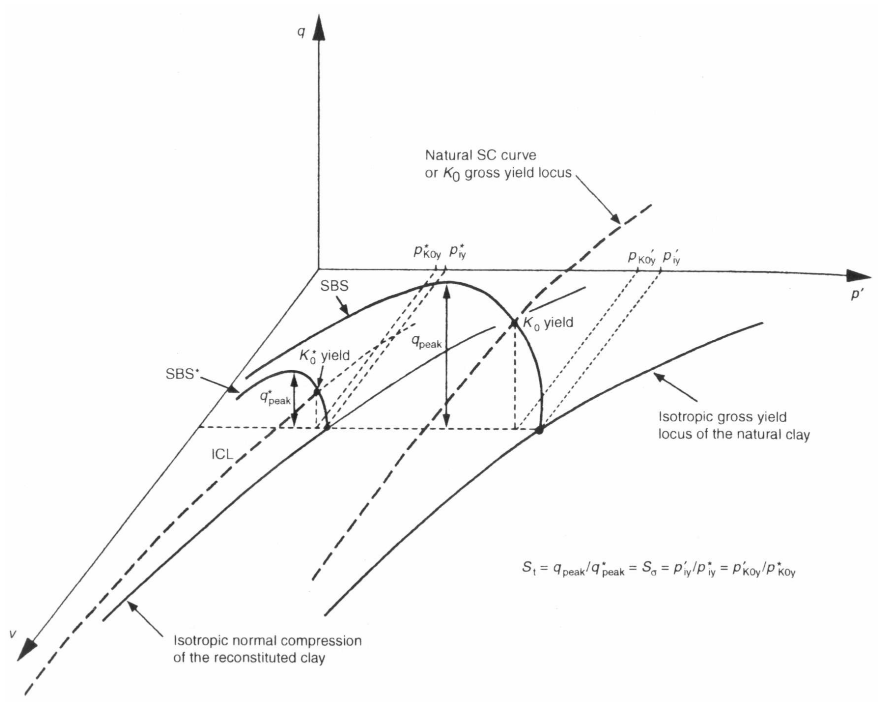
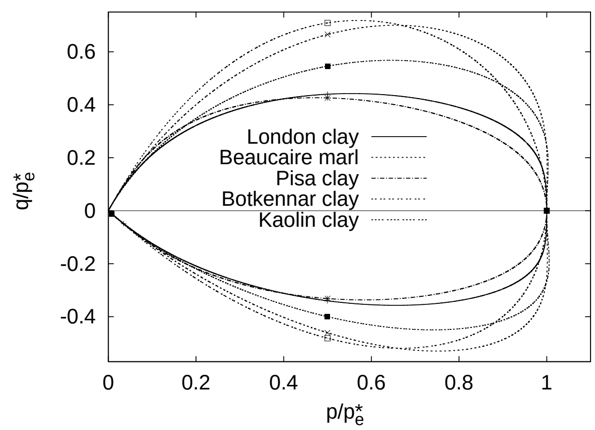


Figure 1: Framework for structured fine-grained materials (Cotecchia and Chandler 2000)



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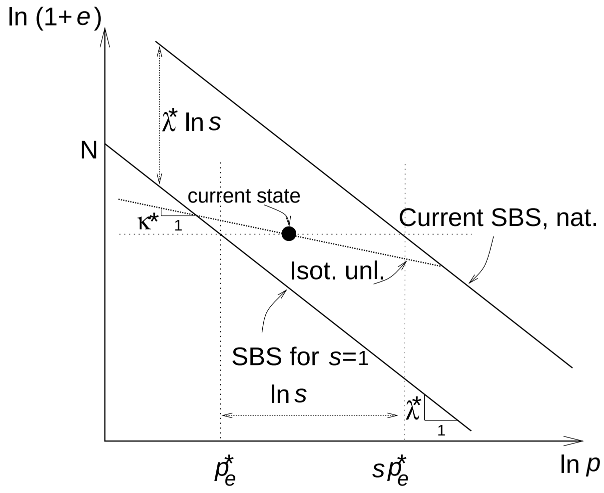

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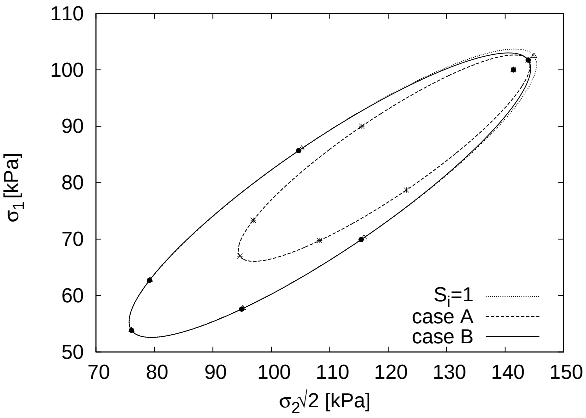

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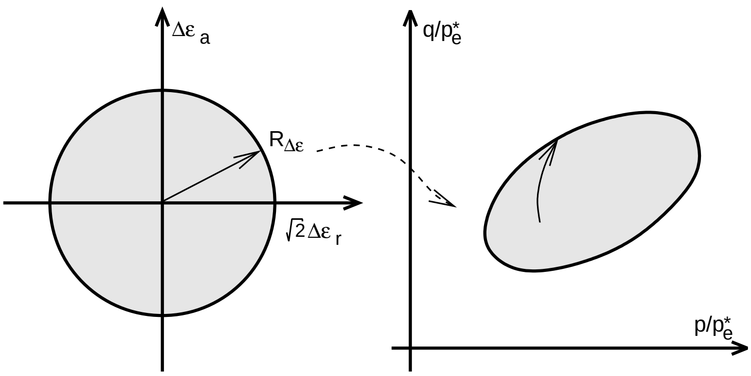

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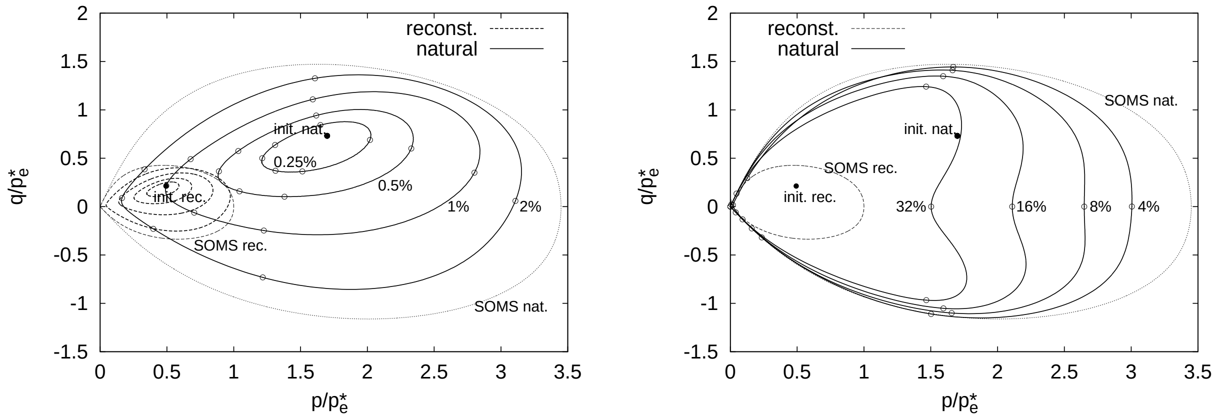

(a) (b)

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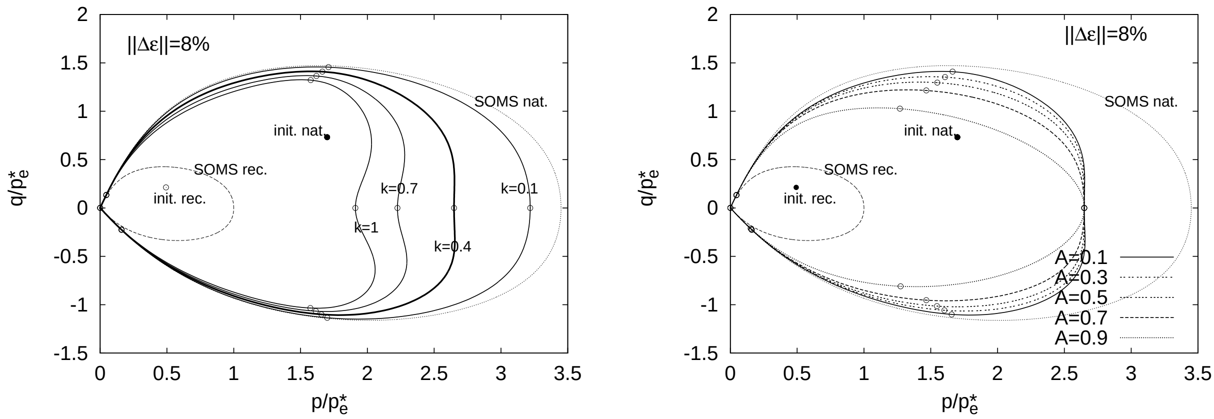

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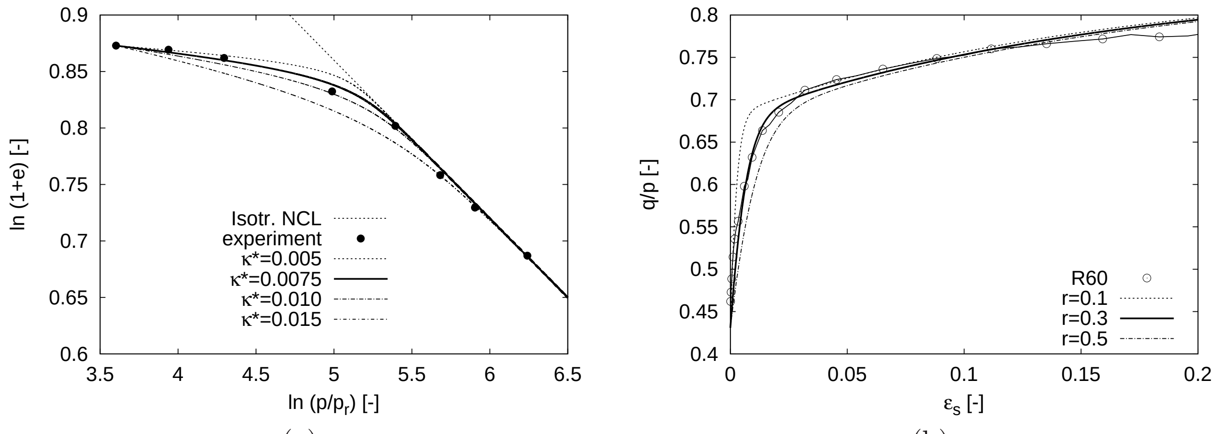

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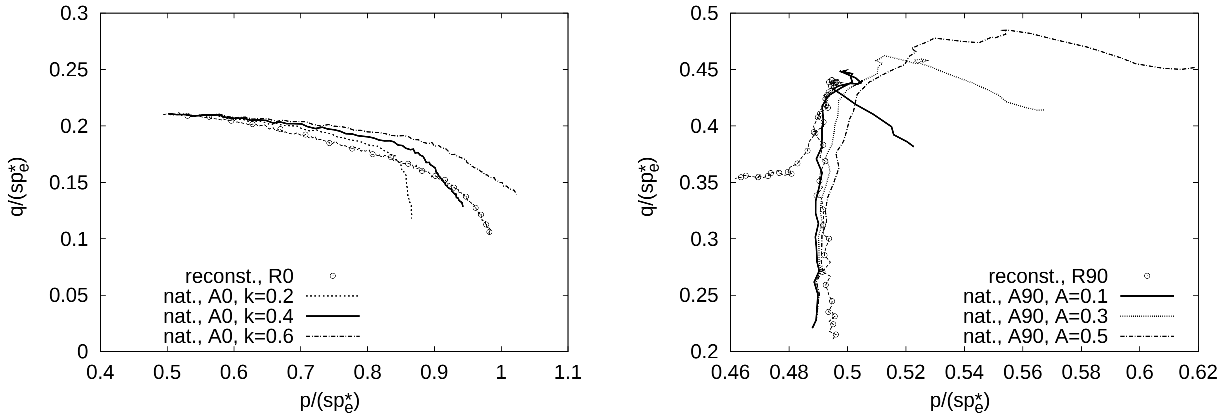

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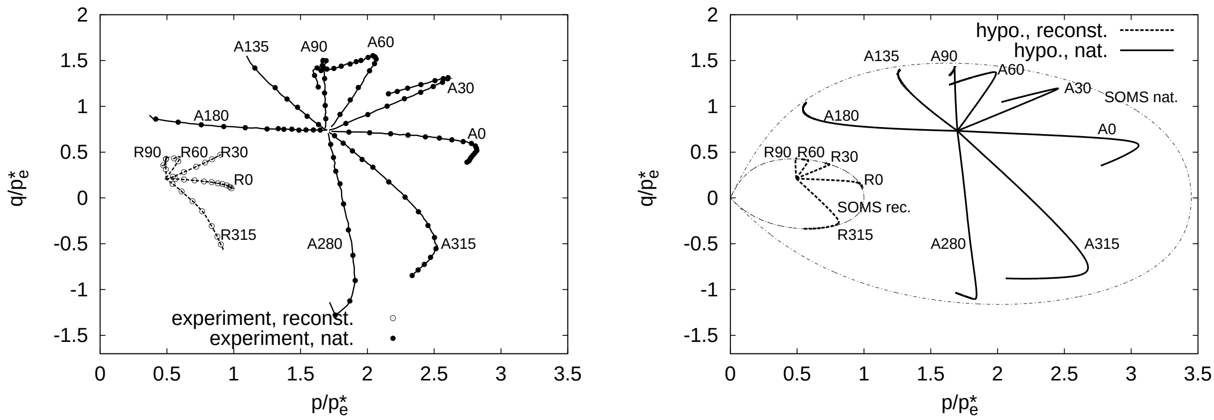

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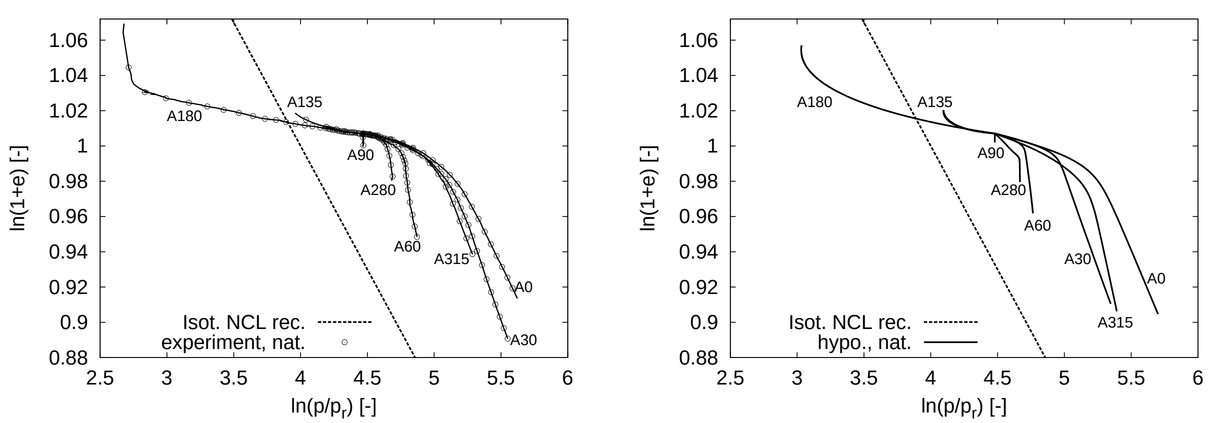

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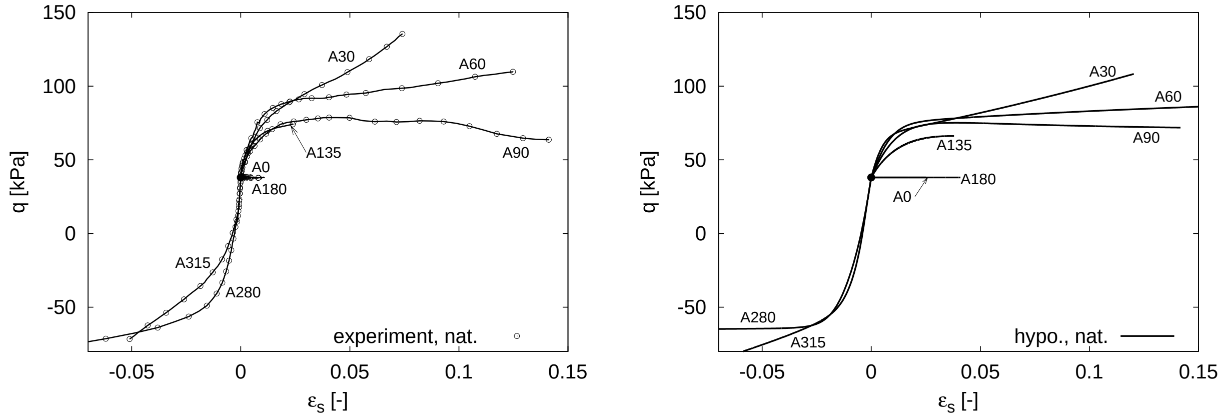

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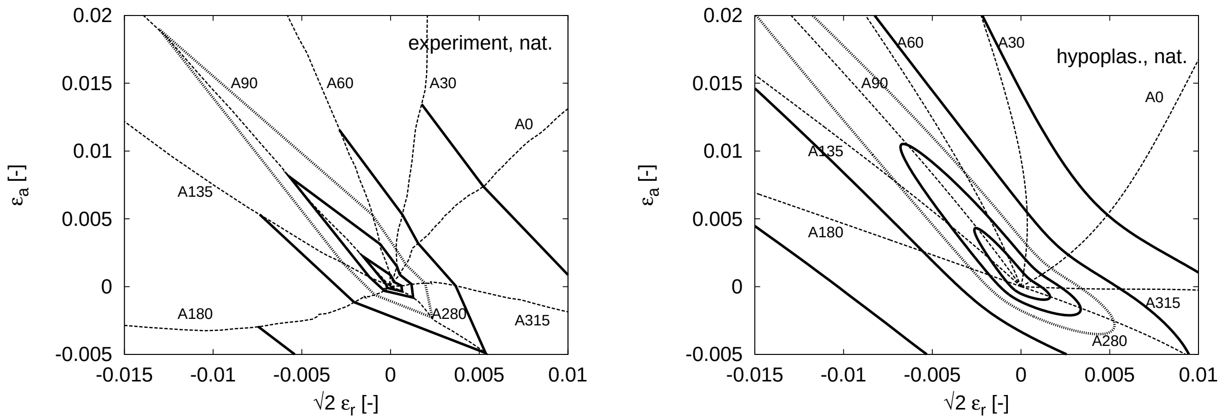

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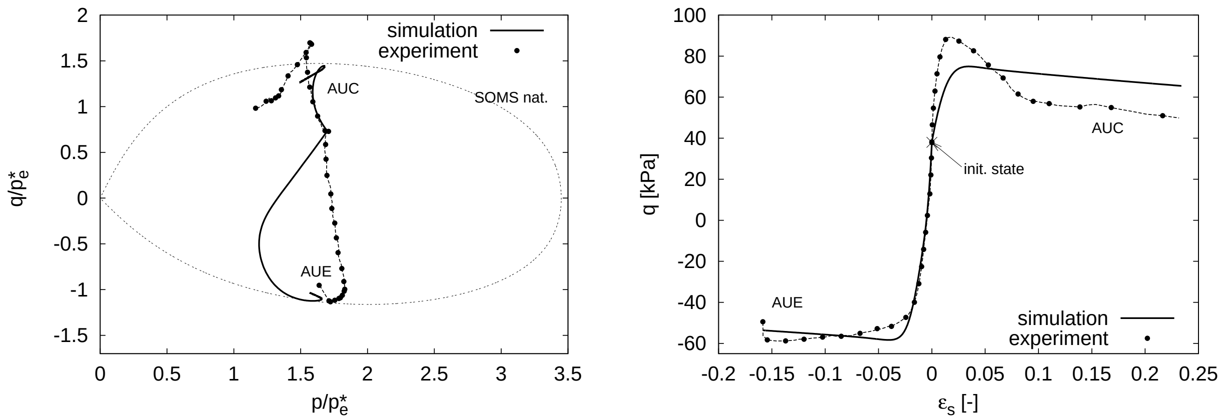

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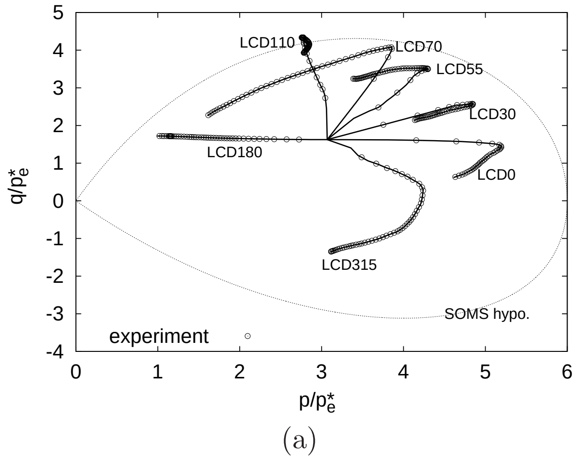

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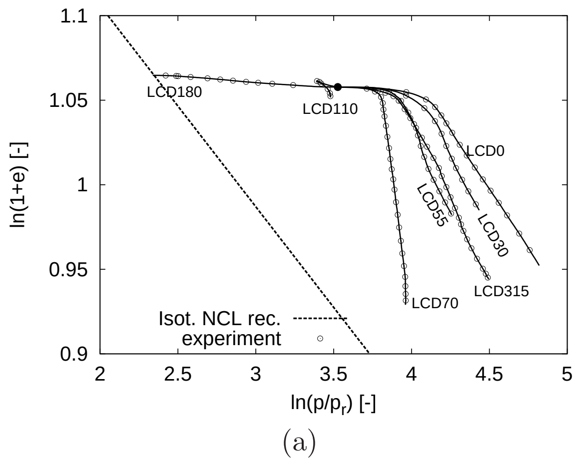

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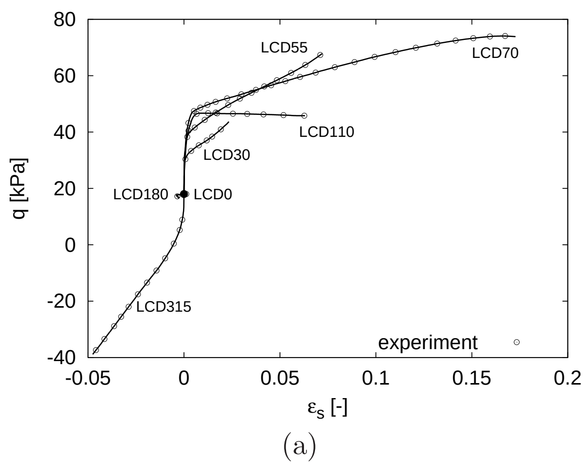

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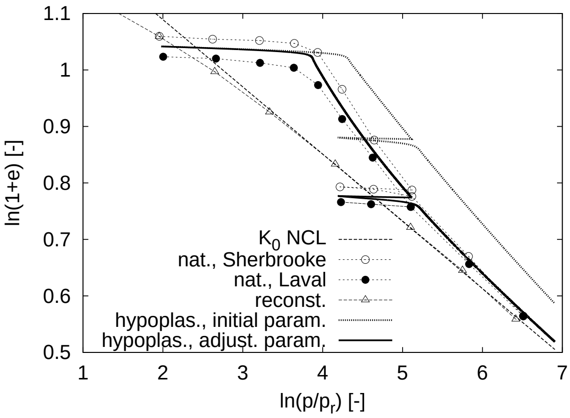

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