Modelling of the collapsible behaviour of unsaturated soils in hypoplasticity

D. Mašín
Charles University, Prague, Czech Republic

N. Khalili
University of New South Wales, Sydney, Australia

ABSTRACT: The paper presents a recently developed constitutive model for unsaturated soils, based on the theory of hypoplasticity and the effective stress principle. The mathematical formulation of the model is outlined and the required state variables and parameters are described. The model is, among other features of unsaturated soil behaviour, capable of predicting collapse upon wetting, a phenomenon that could not be modelled with earlier hypoplastic models. Predictions of wetting-induced collapse agree well with experimental data on statically compacted Pearl clay.

1 INTRODUCTION

Hypoplasticity, a particular class of incrementally non-linear constitutive models, has undergone a notable development during last two decades. Recently, hypoplastic models cover a wide range of geomaterials, such as granular materials, soils with a low friction angle and clays. Procedures to incorporate anisotropy, viscosity, structure and the elastic behaviour in the very small strain range and the effects of recent history are available. To date, however, most contributions on the constitutive modelling of soils using the theory of hypoplasticity have been in the domain of saturated soils. Extension of this class of constitutive models to unsaturated soils is presented in this contribution.

Mašín and Khalili (2007) have recently developed a new hypoplastic model for unsaturated soils. The model is based on the hypoplastic model for clays by Mašín (2005). It is thus, as other advanced hypoplastic models, characterised by the following rate form:

$$\overset{\circ}{\mathbf{T}} = f_s \left(\boldsymbol{\mathcal{L}} : \mathbf{D} + f_d \mathbf{N} \|\mathbf{D}\| \right) \tag{1}$$

where $\overset{\circ}{\mathbf{T}}$ is the objective rate of Cauchy stress tensor, $\mathbf{D}$ is Euler stretching tensor, $\boldsymbol{\mathcal{L}}$ and $\mathbf{N}$ are two constitutive tensors and f_s and f_d are two scalar factors (named *barotropy* and *pyknotropy* factors respectively) that incorporate the influence of mean stress and void ratio. The model by Mašín (2005) is characterised by a low number of parameters and a simple calibration procedure. This advantageous property of the basic model is naturally shared also by its extension for unsaturated soils.

The aim of this contribution is to outline mathematical formulation and basic features of the hypoplastic model for unsaturated soils. The model is then evaluated with respect to experimental data on one characteristic feature of the unsaturated soil behaviour – collapse of the structure caused by wetting. More detailed description and evaluation of the model may be found in Mašín and Khalili (2007).

Throughout this paper, sign conversion of continuum mechanics is considered, i.e. compression is taken as negative.

2 STRESS STATE VARIABLES

Central to the framework presented here is the concept of effective stress which can be defined in the following general form, subject to the solid grains incompressibility constraint (e.g., Bishop 1959)

$$\mathbf{T} = \mathbf{T}^{net} + \mathbf{1}\chi s \tag{2}$$

Stress variables without any superscript ($\mathbf{T}$) denote the effective stress, $\mathbf{T}^{net}$ is the net stress defined as $\mathbf{T}^{net} = \mathbf{T}^{tot} - \mathbf{1}u_a$ and $s = u_a - u_w$ is the matric suction. $\mathbf{T}^{tot}$ is the total stress, u_a is the pore air pressure and u_w is the pore water pressure.

A simple formulation for the effective stress tensor $\mathbf{T}$ based on Eq. (2), which is sufficient for many practical applications, has been put forward by Khalili and Khabbaz (1998) and further evaluated by Khalili et al.

(2004). On the basis of an extensive evaluation of experimental data they proposed the following empirical formulation for χ:

$$\chi = \begin{cases} 1 & \text{for} \quad s \geq s_e \\ \left(\dfrac{s_e}{s}\right)^{\gamma} & \text{for} \quad s < s_e \end{cases} \tag{3}$$

where s_e is the suction value separating saturated from unsaturated states. It is equal to the air entry value for drying processes and the air expulsion value for wetting processes. γ is a material parameter, and it has been shown that for a broad range of different soils it is sufficient to assign $\gamma = 0.55$ (Khalili and Khabbaz 1998). For suctions lower than s_e the effective stress parameter χ is equal to 1, i.e. the soil is saturated and Eq. (2) reduces to the Terzaghi effective stress definition.

Time differentiation of Eq. (2), with the use of (3) and taking into account rigid body rotations, imply the following formulation of the objective rate of the effective stress

$$\mathring{\mathbf{T}} = \mathring{\mathbf{T}}^{net} + \mathbf{1}(1-\gamma)\chi\dot{s} \tag{4}$$

In addition to the effective stress tensor $\mathbf{T}$, suction s is considered as a state variable that quantifies the stiffening effect of the water menisci.

3 HYPOPLASTIC MODEL FOR UNSATURATED SOILS

In this section, the hypoplastic model for unsaturated soils proposed recently by Mašín and Khalili (2007) will be presented. The basic aim of the derivations in this section is to demonstrate a conceptual way to incorporate the behaviour of unsaturated soils into hypoplasticity. The particular formulation adopted is very simple, but it may be readily modified by using the general rules outlined in this section.

3.1 *Model for constant suction*

The overall mechanical response of a soil element is controlled by the effective stress tensor. Suction influences the effective stress and, in addition, it increases normal forces at interparticle contacts and thus acts as a quantity that increases the overall stability of the soil structure. In terms of the critical state soil mechanics, it increases the size of the state boundary surface (SBS), in a similar manner to bonding between soil particles in saturated cemented materials. State boundary surface is defined as a boundary of all possible states of a soil element in the stress vs. void ratio space.

The incorporation of structure into hypoplastic model has been discussed in detail by Mašín (2007). In this context, the size of the SBS for unsaturated soils is controlled by the isotropic virgin compression line with the formulation according to Butterfield (1979)

$$\ln(1+e) = N(s) - \lambda^*(s) \ln \frac{p}{p_r} \tag{5}$$

where e is the void ratio, which is considered as a state variable, and $p_r = 1$ kPa is a reference stress. Quantities $N(s)$ and $\lambda^*(s)$ define the position and the slope of the isotropic virgin compression line in the $\ln(p/p_r)$ vs. $\ln(1+e)$ plane for given suction s.

For the evaluation of model predictions through this paper, we assume for $\ln(s/s_e) > 0$ (unsaturated state) the following simple logarithmic dependency of $N(s)$ and $\lambda^*(s)$ on s:

$$N(s) = N + n \ln\left(\frac{s}{s_e}\right) \tag{6}$$

$$\lambda^*(s) = \lambda^* + l \ln\left(\frac{s}{s_e}\right) \tag{7}$$

where the quantities n and l represent two additional soil parameters. For $\ln(s/s_e) < 0$ (saturated state) $N(s) = N$ and $\lambda^*(s) = \lambda^*$. It is, however, emphasized that the general formulation of the model can accommodate any other more complex relationships between $N(s)$, $\lambda^*(s)$ and s.

Mašín (2007) demonstrated that incorporation of variable virgin compressibility and the intercept $N(s)$ into the hypoplastic model requires a modification of both barotropy and pyknotropy factors f_s and f_d in (1), which are now calculated in terms of $N(s)$ and $\lambda^*(s)$. The respective expressions are given in Mašín and Khalili (2007).

3.2 *Incorporation of wetting-induced collapse at normally consolidated states*

When an unsaturated soil with an initially open structure is subjected to a decreasing suction, the reduction in the normal forces acting at the inter-particle contacts may result in a situation in which the structure, for the given effective stress $\mathbf{T}$ and void ratio e, is no longer stable, and thus it collapses. This phenomenon, referred to as a wetting-induced collapse, cannot be modelled with the model for structured clays (Mašín 2007), as $\mathring{\mathbf{T}} = \mathbf{0}$ implies $\mathbf{D} = \mathbf{0}$ (see Eq. (1)), i.e. no deformation of the soil skeleton can be predicted for variable suction with constant effective stress.

In the context of the critical state soil mechanics, all admissible states of a soil element are bounded by the SBS. As the hypoplastic model from Sec. 3.1 predicts constant void ratio sections through the SBS of the same shape (see Mašín and Herle (2005)), it is advantageous to study collapse due to wetting in the stress space normalised by the size of the SBS for current

e. This size is quantified by the Hvorslev equivalent pressure p_e, implied by Eq. (5).

Mašín and Khalili (2007) have shown, that normalisation with respect to p_e allows us to derive the following expression that ensures consistency of the model predictions with the SBS of suction-dependent size:

$$\mathring{\mathbf{T}} = f_s\left(\boldsymbol{\mathcal{L}} : \mathbf{D} + f_d \mathbf{N}\|\mathbf{D}\|\right) + \mathbf{H} \tag{8}$$

where $\mathbf{H}$ is a new term given by

$$\mathbf{H} = \frac{\mathbf{T}}{p_e}\frac{\partial p_e}{\partial s}\dot{s} \tag{9}$$

From the expression for the Hvorslev equivalent pressure p_e follows

$$\mathbf{H} = \frac{\mathbf{T}}{\lambda(s)}\left[\frac{\partial N(s)}{\partial s} - \frac{\partial \lambda(s)}{\partial s}\ln\frac{p_e}{p_r}\right]\dot{s} \tag{10}$$

3.3 *Model for any state of overconsolidation*

The model from Sec. 3.2 may be used for constant value of suction ($\dot{s} = 0$) and for wetting at normally consolidated states (states at the SBS). The following assumptions are utilised to extend Eq. (8) for arbitrary (physically admissible, i.e. inside the SBS) states and arbitrary loading conditions:

1. As suction controls stability of inter-particle contacts, increasing suction under constant *effective* stress imposes no deformation of soil skeleton.
2. The more open the soil structure, the larger the inter-particle contact shear forces and therefore the greater the number of inter-particle contact slips under wetting at constant effective stress.

To reflect these two assumptions, the rate formulation of the model is written as

$$\mathring{\mathbf{T}} = f_s\left(\boldsymbol{\mathcal{L}} : \mathbf{D} + f_d \mathbf{N}\|\mathbf{D}\|\right) + f_u\mathbf{H} \tag{11}$$

with

$$\mathbf{H} = \frac{\mathbf{T}}{\lambda(s)}\left[\frac{\partial N(s)}{\partial s} - \frac{\partial \lambda(s)}{\partial s}\ln\frac{p_e}{p_r}\right]\langle\dot{s}\rangle \tag{12}$$

where the operator $\langle x\rangle$ denotes positive part of any scalar function x and f_u is a new pyknotropy factor controlling tendency of the soil structure to collapse upon wetting.

The factor f_u must be equal to unity for states at the SBS (in that case the structure is as open as possible and collapse is controlled by $\mathbf{H}$ only) and $f_u \to 0$ for $OCR \to \infty$ (no wetting-induced inter-particle slippage occurs in highly overconsolidated soil). The following expression for the factor f_u satisfying these requirements is proposed:

$$f_u = \left(\frac{p}{p^{SBS}}\right)^m \tag{13}$$

where p^{SBS} is the effective mean stress at the SBS corresponding to the current stress state $\mathbf{T}/\operatorname{tr}\mathbf{T}$ and current void ratio e and m is a model parameter controlling the influence of overconsolidation on the wetting-induced collapse. Eq. (13) is demonstrated graphically in Fig. 1. Clearly, value of the parameter m controlls dependency between collapse of structure and distance of the current state from the SBS. Note that basic elasto-plastic models based on suction hardening concept imply $m \to \infty$ (collapse at the yield surface only).

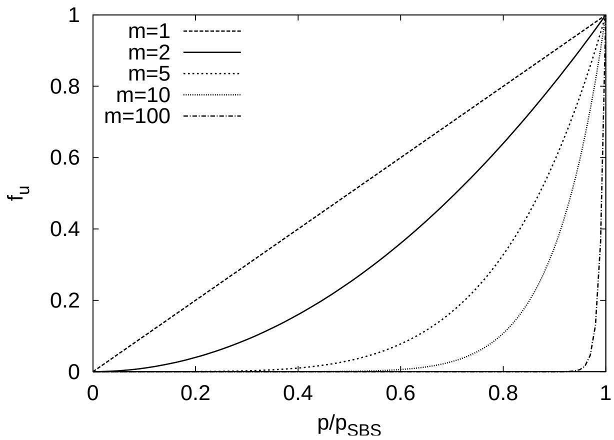

Figure 1. The influence of the parameter m on the value of suction hardening pyknotropy factor f_u.

It may be shown from the definition of the pyknotropy factor f_d of the basic hypoplastic model and using rules derived by Mašín and Herle (2005) that

$$f_u = \left[f_d\|f_s\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|\right]^{m/\alpha} \tag{14}$$

where the fourth-order tensor $\boldsymbol{\mathcal{A}}$ is given by

$$\boldsymbol{\mathcal{A}} = f_s\boldsymbol{\mathcal{L}} - \frac{1}{\lambda^*(s)}\mathbf{T}\otimes\mathbf{1} \tag{15}$$

4 WETTING-INDUCED STRAIN RATE

Wetting of normally consolidated soil at anisotropic stress state causes in addition to volumetric collapse development of shear strains (Sun et al. 2004, 2007). Eq. (8) allows us to derive an expression for the direction of stretching implied by wetting at constant effective stress for states at the SBS (see Mašín and Khalili (2007)).

$$\vec{\mathbf{D}} = -\frac{\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}}{\|\boldsymbol{\mathcal{A}}^{-1} : \mathbf{N}\|} \tag{16}$$

where the fourth-order tensor $\boldsymbol{\mathcal{A}}$ is given by Eq. (15).

Eq. (16) implies purely deviatoric strain rate at the critical state and purely volumetric strain rate at the isotropic stress state. Direction of the strain increment vector for different stress obliquities is graphically demonstrated in Fig. 2, together with the shape of the bounding surface for Pearl clay parameters (Tab. 1), evaluated by Mašín and Khalili (2007). It is clear that the strain increment vector is not perpendicular to the SBS (in terms of elasto-plasticity, neglecting the effects of elastic strains, this would be implied by a non-associated flow rule).

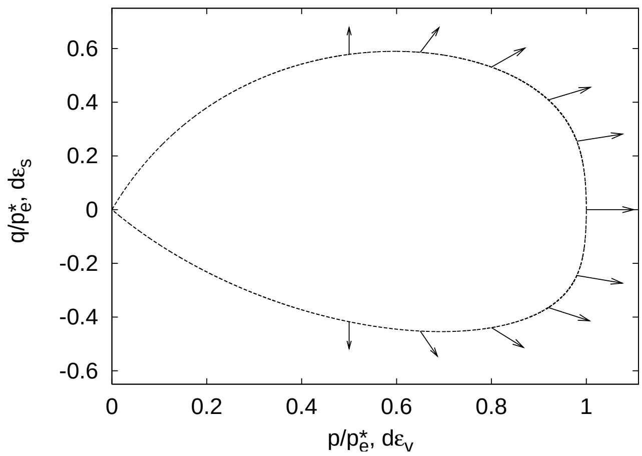

Figure 2. Direction of strain rate tensor induced by wetting at constant effective stress for Pearl clay parameters.

rameter m from Eq. (14). The higher the value of m, the closer to the SBS the volumetric collapse takes place. The value of $m = 2$ has been considered as a suitable value to represent Pearl clay behaviour. Calibration of all other model parameters for Pearl clay (Tab. 1) is detailed in Mašín and Khalili (2007).

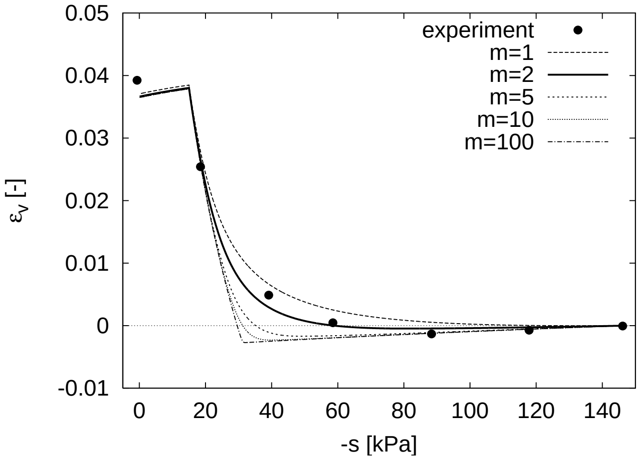

Figure 3. s vs. ϵ_v relationship for wetting of slightly overconsolidated soil at constant net stress.

5 PREDICTING THE COLLAPSIBLE BEHAVIOUR OF UNSATURATED SOILS

Thorough evaluation of the hypoplastic model for unsaturated soils is presented in Mašín and Khalili (2007). It contains response to drying and wetting paths of soil specimens at isotropic and anisotropic stress states and response to constant suction shear tests and isotropic loading tests at different suction levels. Tests on five different soils performed in different soil mechanics laboratories are used for evaluation. Due to the limited space, in this paper we restrict the model evaluation to tests at the isotropic and anisotropic stress state under constant and decreasing suction. The response to wetting paths is with respect to hypoplastic modelling the most important to study, as in this case the new terms $\mathbf{H}$ and f_u are activated.

The model is evaluated by means of experimental data on statically compacted Pearl clay by Sun et al. (2004, 2007). Pearl clay is a moderate plasticity soil with very little expansive clay minerals. The first set of experimental data consist of tests on soil specimens that have been isotropically compressed at constant suction -147 kPa to different mean net stress levels (49, 98, 196, 392 and 588 kPa). At this stage, the specimens were wetted at constant net stress and suction was decreased to zero. Some of the specimens were further compressed at zero suction to the mean net stress 588 kPa.

Figure 3 shows response to wetting tests at the highest apparent overconsolidation ratio (the test where wetting took place at $p^{net} = 49$ kPa) and predictions by the model with different values of the parameter m from Eq. (14) — see above.

Table 1. *Parameters of the hypoplastic model for Pearl clay (calibrated using data from Sun et al. (2004)).*

φ_c	λ^*	κ^*	N	r
29°	0.05	0.005	1.003	0.5
n	l	m	s_e [kPa]	
0.164	0.024	2	-15	

Figure 4 shows graphs of the constant suction isotropic compression tests and constant net stress wetting tests replotted in the effective stress space. Predictions are in a good agreement with the experimental results, the model predicts correctly both the constant suction and wetting parts of the experiments. In the wetting tests at the lower net mean stresses, the experiments show the initial decrease of the effective stress with very small change of void ratio. This aspect of the observed soil behaviour, which is progressively less pronounced with decreasing apparent OCR, can be modelled correctly by the proposed model thanks to the new pyknotropy factor f_u.

Results of the wetting parts of the experiments from Fig. 4 are plotted in the suction vs. volumetric strain plane in Fig. 5. The model predicts correctly the qualitative influence of the net mean stress on the volumetric behaviour. When the soil is wetted at low net mean stress (49 kPa), it first swells and only after the state gets closer to the state boundary surface the structure starts to collapse. On the other hand, specimens wetted at higher net mean stresses (i.e. at lower apparent OCRs) collapse since the beginning of the

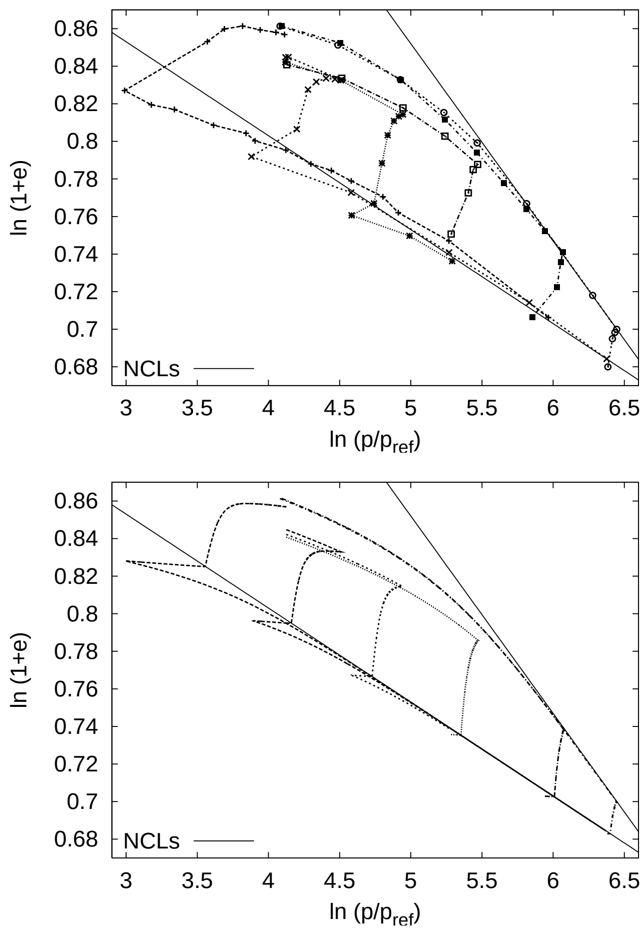

Figure 4. Isotropic compression tests at constant suction and wetting tests at constant net stress by Sun et al. (2007) replotted in the effective stress space (top) and predictions by the proposed model (bottom).

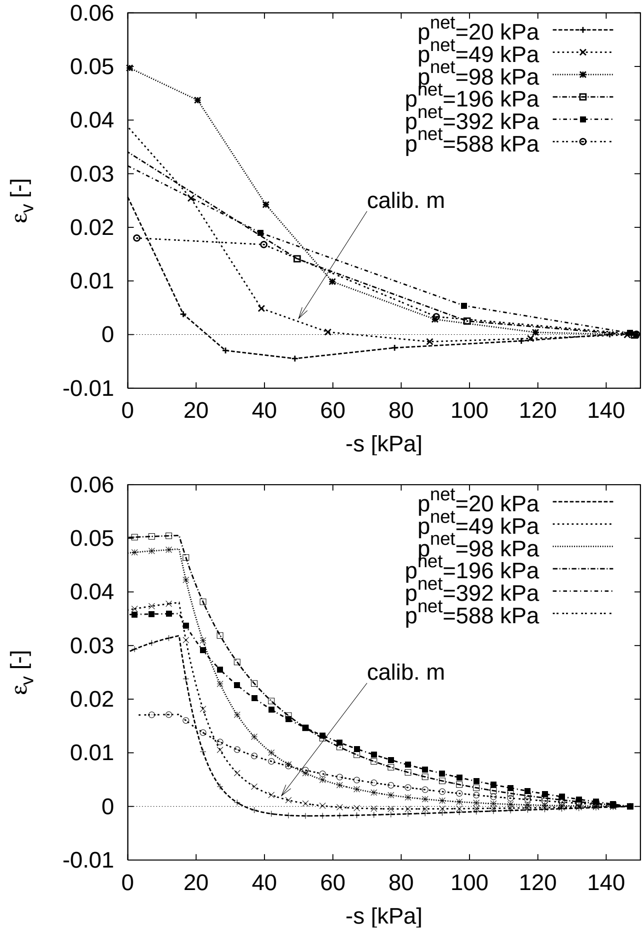

Figure 5. Wetting tests at constant isotropic net stress by Sun et al. (2007) plotted in s vs. ϵ_v plane (top) and predictions by the proposed model (bottom).

wetting test. This aspect of the soil behaviour is predicted correctly thanks to the proposed formulation for the factor f_u. The experiments show the lowest collapsible strains for the wetting at the highest net mean stress (588 kPa). Correct predictions of the final value of the volumetric strains after collapse are achieved thanks to the converging normal compression lines of the saturated and unsaturated soils (Fig. 4), i.e. thanks to $l > 0$ (Eq. (6)). The predicted shape of the wetting path in the s vs. ϵ_v plane is controlled by the factor f_u (for the initially apparently overconsolidated specimens) and by the interpolation function for the quantities $N(s)$ and $\lambda^*(s)$ (Eq. (6)). Good agreement between experimental data and model predictions also for wetting at higher net mean stresses (where the factor f_u takes a constant value equal to 1) suggests that the logarithmic interpolation adopted is suitable to represent the actual soil behaviour.

The second set of experimental data allows us to investigate the influence of the stress anisotropy on the wetting-induced collapse behaviour. The specimens were, after isotropic compression at constant suction $s = -147$ kPa to mean net stress $p^{net} = 196$ kPa, subjected to constant suction and constant net mean stress shear tests up to a target principal net stress ratio $R = T_a^{net}/T_r^{net}$, where T_a^{net} and T_r^{net} are the axial and radial net stresses. At this stage, suction was decreased to zero under constant net stress, and finally the shear test continued under constant mean net stress and $s = 0$ kPa to failure. The specimens had approximately equal initial void ratios (initial apparent $OCRs$) and they were wetted at different values of the ratio R (1.5, 2 and 2.5).

Figure 6 shows the results of the three constant net mean stress shear tests in the axial strain vs. principal net stress ratio plane. The corresponding radial strains are in Fig. 7. Correct predictions of the constant suction parts of the tests demonstrate the predictive capabilities of the basic hypoplastic model, which predicts the non-linear soil behaviour with gradual decrease of the shear stiffness. In the wetting parts of the tests, the model predicts significant increase of the collapse axial strains and of the negative radial strains at higher ratios R. The good quantitative agreement for both ϵ_a and ϵ_r demonstrates adequate modelling of the wetting-induced collapse strain rate direction. The analytical expression for this direction has been (for constant effective stress) derived in Sec. 4, see Fig. 2 for Pearl clay parameters.

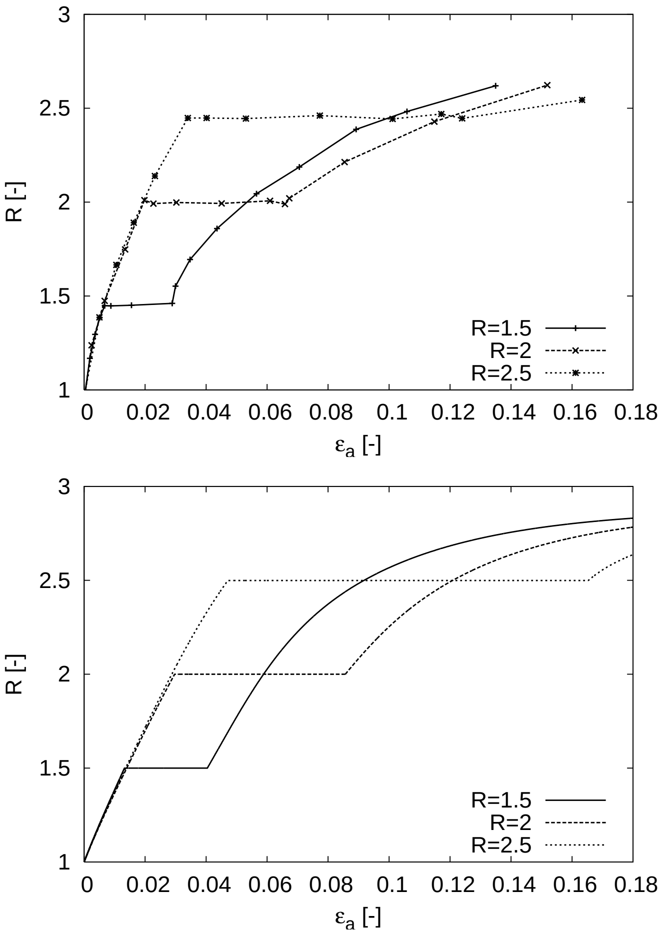

Figure 6. Constant net mean stress shear tests and constant R wetting tests by Sun et al. (2007) plotted in ϵ_a vs. $R = T_a/T_r$ plane (top) and predictions by the proposed model (bottom).

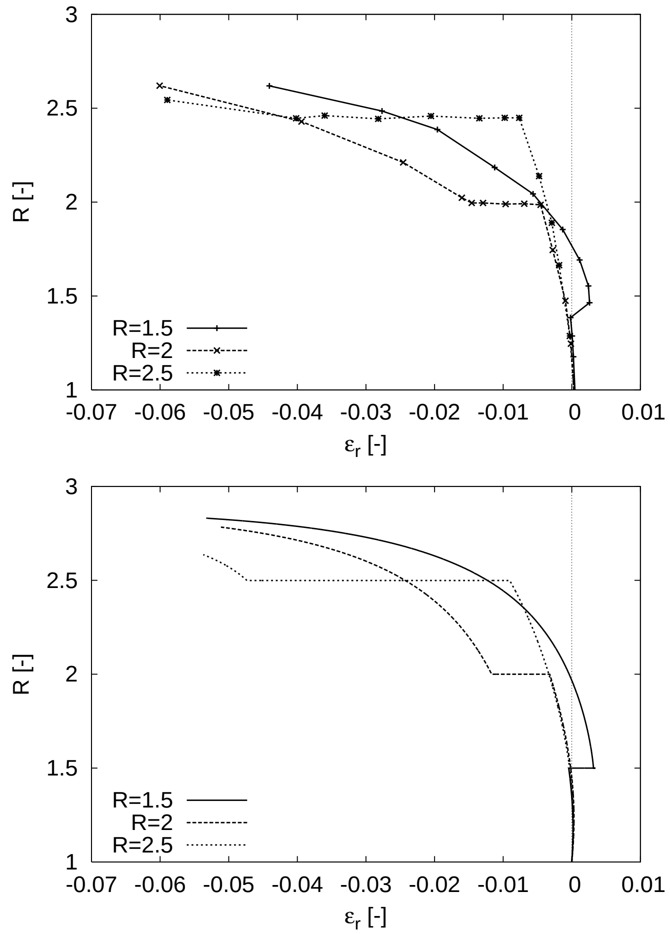

Figure 7. Constant net mean stress shear tests and constant R wetting tests by Sun et al. (2007) plotted in ϵ_a vs. ϵ_r plane (top) and predictions by the proposed model (bottom).

6 CONCLUDING REMARKS

A recently developed constitutive model for unsaturated soils is presented in the paper. The model is based on the theory of hypoplasticity, it is thus capable of predicting pre- and post-peak non-linear deformation behaviour of unsaturated soils, and the variation of the soil stiffness with loading direction - important aspects absent from many of the current constitutive models proposed for the behaviour of unsaturated soils.

A specific feature of unsaturated soil behaviour – collapse of the structure induced by wetting – can be predicted thanks to the factors **H** and f_u, novel to hypoplasticity. Predictions of the wetting-induced collapse, presented in this paper, agree well with experimentally observed behaviour.

7 ACKNOWLEDGEMENT

The first author acknowledges the financial support by the research grants GAAV IAA200710605, GACR 103/07/0678 and MSM0021620855.

REFERENCES

Bishop, A. W. (1959). The principle of effective stress. *Teknisk Ukeblad 106*(39), 859–863.

Butterfield, R. (1979). A natural compression law for soils. *Géotechnique 29*(4), 469–480.

Khalili, N., F. Geiser, and G. E. Blight (2004). Effective stress in unsaturated soils: review with new evidence. *International Journal of Geomechanics 4*(2), 115–126.

Khalili, N. and M. H. Khabbaz (1998). A unique relationship for χ for the determination of the shear strength of unsaturated soils. *Géotechnique 48*(2), 1–7.

Mašín, D. (2005). A hypoplastic constitutive model for clays. *International Journal for Numerical and Analytical Methods in Geomechanics 29*(4), 311–336.

Mašín, D. (2007). A hypoplastic constitutive model for clays with meta-stable structure. *Canadian Geotechnical Journal 44*(3), 363–375.

Mašín, D. and I. Herle (2005). State boundary surface of a hypoplastic model for clays. *Computers and Geotechnics 32*(6), 400–410.

Mašín, D. and N. Khalili (2007). A hypoplastic model for mechanical response of unsaturated soils. *International Journal for Numerical and Analytical Methods in Geomechanics (submitted)*.

Sun, D. A., H. Matsuoka, and Y. F. Xu (2004). Collapse behaviour of compacted clays in suction-controlled triaxial tests. *Geotechnical Testing Journal 27*(4), 362–370.

Sun, D. A., D. Sheng, and Y. F. Xu (2007). Collapse behaviour of unsaturated compacted soil with different initial densities. *Canadian Geotechnical Journal 44*(6), 673–686.