CLoE model modified to predict the behaviour of normally compressed clays

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ABSTRACT: The paper presents a version of the CLoE model developed for predictions of behaviour of normally-compressed fine grained soils. The model enables direct calibration of isotropic loading and unloading moduli and introduces modifications into analytical formulations of basic paths in order to take into account the re-evaluated consistency condition at the isotropic stress state. The intention of the paper is to demonstrate the flexibility of the formulation of the CLoE model and to highlight some basic properties of hypoplastic constitutive models in general.

1 Introduction

The CLoE model (Chambon, 1989; Chambon et al., 1994) was developed in Grenoble in 1990's as a particular type of hypoplastic constitutive models with the aim of predicting the behaviour of granular materials at medium to large strain levels. Explicit formulation of the localisation criterion, with the possibility of an independent calibration of the out-of-axis shear modulus, and presence of a single surface in the stress space, which bounds all admissible stress states, are specific features of the model, distinguishing it from other hypoplastic models. The constitutive equation is given, in rate-form, by:

$$\underline{\dot{\sigma}} = \underline{\underline{\mathbf{A}}}\underline{\dot{\varepsilon}} + \underline{\mathbf{b}} \left\| \underline{\dot{\varepsilon}} \right\| \tag{1}$$

The first term on the right-hand side represents an incrementally linear behaviour, while the latter accounts for incremental non-linearity via a linear dependence on the norm of the strain rate tensor. To keep the formulation as simple as possible, the set of state variables for the material is limited to the Cauchy stress tensor.

The two constitutive tensors $\underline{\underline{A}}$ and $\underline{b}$ appearing in (1) are homogeneous functions of degree one of the stress tensor, for which no explicit expression is assumed. Rather, $\underline{\underline{A}}$ and $\underline{b}$ are obtained via an interpolation procedure based on the assigned material responses at some suitably defined image points, located along special loading paths (*basic paths*). These are selected among those stress-paths that are experimentally accessible by means of conventional laboratory tests, namely drained triaxial compression and extension paths, isotropic loading, and "pseudo-isotropic" compression paths (for details see Chambon et al., 1994).

Because the isotropic unloading is not a basic path, it is not possible to calibrate independently the bulk moduli in isotropic loading and unloading. While the unloading bulk modulus is predicted relatively correctly for granular materials, for normally compressed clays, which are very soft in isotropic compression, is significantly underpredicted (Mašín et al., 2005).

In the *modified* model, presented in the paper, the isotropic unloading is introduced as an additional basic path. In order to retain the basic features of the CLoE model, the set of state variables is restricted to the Cauchy stress. For this reason, the presented model can not be considered as suitable for problems, where larger changes in void ratio occur. The intention of the paper is rather to demonstrate the flexibility of the formulation of the CLoE model and to highlight some basic properties of hypoplastic constitutive models in general.

2 Consistency condition at the isotropic stress state

As discussed in detail by Chambon et al. (1994), the response of the model along all basic paths must merge at the isotropic stress state and must be consistent with the pre-defined van Eekelen limit surface. In order to introduce the isotropic unloading as an additional basic path, the consistency condition at the isotropic stress state must be re-evaluated. The constitutive tensors $\underline{\underline{A}}$ and $\underline{b}$ for the isotropic stress state in the principal stress space take the form:

$$\underline{\underline{\mathbf{A}}} = \begin{bmatrix} a & d & d & & & \\ d & a & d & & & \\ d & d & a & & & \\ & & & a-d & & \\ & & & & a-d & \\ & & & & & a-d \end{bmatrix} \tag{2}$$

$$\underline{\mathbf{b}} = \begin{bmatrix} k & k & k & 0 & 0 & 0 \end{bmatrix}^T \tag{3}$$

with the three independent moduli: *a*, *d* and *k*. The *modified* CLoE model assumes *four* basic paths, for which the consistency at the isotropic stress state must be evaluated:

- triaxial axisymmetric drained compression
- triaxial axisymmetric drained extension
- isotropic loading
- isotropic unloading (additional basic path)

Because the CLoE constitutive equation is positively homogeneous of degree one with respect to stress, which is suitable for modelling fine-grained soils (Mašín, 2004), it is possible to study the consistency condition in the normalised stress space. Normalised stress and strain increment vectors for different tests may be written (using soil mechanics sign convention – compression positive) as:

1. triaxial axisymmetric drained compression

$$\underline{\dot{\sigma}} = \{1,0,0,0,0,0\}^T \qquad \underline{\dot{\varepsilon}} = \{\dot{\varepsilon}_{aC}, \dot{\varepsilon}_{lC}, \dot{\varepsilon}_{lC}, 0,0,0\}^T \tag{4}$$

2. triaxial axisymmetric drained extension

$$\underline{\dot{\sigma}} = \{-1,0,0,0,0,0\}^T \qquad \underline{\dot{\varepsilon}} = \{\dot{\varepsilon}_{aE}, \dot{\varepsilon}_{lE}, \dot{\varepsilon}_{lE}, 0,0,0\}^T \tag{5}$$

3. isotropic loading

$$\underline{\dot{\sigma}} = \{1,1,1,0,0,0\}^T \qquad \underline{\dot{\varepsilon}} = \{\dot{\varepsilon}_{ic}, \dot{\varepsilon}_{ic}, \dot{\varepsilon}_{ic}, 0,0,0\}^T \tag{6}$$

4. isotropic unloading

$$\underline{\dot{\sigma}} = \{-1,-1,-1,0,0,0\}^T \qquad \underline{\dot{\varepsilon}} = \{\dot{\varepsilon}_{id},\dot{\varepsilon}_{id},\dot{\varepsilon}_{id},0,0,0\}^T \tag{7}$$

It is possible to write *six* equations of consistency at the isotropic stress state for these tests, which follow from Eqs. (1-3) and (4-7):

1. triaxial axisymmetric drained compression

$$1 = a\left(\dot{\varepsilon}_{aC} + kR_C\right) + 2d\left(\dot{\varepsilon}_{lC} + kR_C\right) \tag{8}$$

$$0 = a\left(\dot{\varepsilon}_{lC} + kR_C\right) + d\left(\dot{\varepsilon}_{aC} + \dot{\varepsilon}_{lC} + 2kR_C\right) \tag{9}$$

with

$$R_C = \sqrt{\dot{\varepsilon}_{aC}^2 + 2\dot{\varepsilon}_{lC}^2} \tag{10}$$

2. triaxial axisymmetric drained extension

$$-1 = a\left(\dot{\varepsilon}_{aE} + kR_E\right) + 2d\left(\dot{\varepsilon}_{lE} + kR_E\right) \tag{11}$$

$$0 = a\left(\dot{\varepsilon}_{lE} + kR_E\right) + d\left(\dot{\varepsilon}_{aE} + \dot{\varepsilon}_{lE} + 2kR_E\right) \tag{12}$$

with

$$R_E = \sqrt{\dot{\varepsilon}_{aE}^2 + 2\dot{\varepsilon}_{lE}^2} \tag{13}$$

3. isotropic loading

$$1 = (a+2d)\left(k\sqrt{3}+1\right)\dot{\varepsilon}_{ic} \tag{14}$$

4. isotropic unloading

$$1 = (a+2d)\left(k\sqrt{3}-1\right)\dot{\varepsilon}_{id} \tag{15}$$

For clarity, it is useful to re-write these equations in terms of normalized Young moduli, which are defined by:

$$\dot{\varepsilon}_{aC} = \frac{1}{E_C} \quad \dot{\varepsilon}_{lC} = \frac{-\nu_C}{E_C} \quad \dot{\varepsilon}_{aE} = \frac{-1}{E_E}$$

$$\dot{\varepsilon}_{lE} = \frac{\nu_E}{E_E} \quad \dot{\varepsilon}_{ic} = \frac{1}{E_{ic}} \quad \dot{\varepsilon}_{id} = \frac{-1}{E_{id}} \tag{16}$$

and Poisson ratios in drained triaxial compression (ν_C) and extension (ν_E), defined as ratios of radial and axial strain rates. Substituting the terms from Eq. (16) into Eqs (8-15) we get:

$$1 = \frac{a}{E_C}\left(1 + k\sqrt{1+\nu_C^2}\right) + \frac{2d}{E_C}\left(-\nu_C + k\sqrt{1+\nu_C^2}\right) \tag{17}$$

$$0 = \frac{a}{E_C}\left(-\nu_C + k\sqrt{1+\nu_C^2}\right) + \frac{d}{E_C}\left(1 - \nu_C + 2k\sqrt{1+\nu_C^2}\right) \quad (18)$$

$$-1 = \frac{a}{E_E}\left(-1 + k\sqrt{1+\nu_E^2}\right) + \frac{2d}{E_E}\left(\nu_E + k\sqrt{1+\nu_E^2}\right) \quad (19)$$

$$0 = \frac{a}{E_E}\left(\nu_E + k\sqrt{1+\nu_E^2}\right) + \frac{d}{E_E}\left(-1 + \nu_E + 2k\sqrt{1+\nu_E^2}\right) \quad (20)$$

$$1 = (a+2d)\left(k\sqrt{3}+1\right)\frac{1}{E_{ic}} \quad (21)$$

$$1 = (a+2d)\left(k\sqrt{3}-1\right)\frac{-1}{E_{id}} \quad (22)$$

Equations (17-22) constitute a set of 6 equations with 9 unknowns (*a, d, k,* E_C, ν_C, E_E, ν_E, E_{ic} and E_{id}). It is therefore possible to prescribe values of three of these unknowns. The standard version of the CLoE model prescribes E_C, ν_C and E_{ic}. In order to improve predictions by the CLoE model for normally compressed clays, it has been decided to prescribe normalized Young modulus in isotropic unloading (E_{id}), instead of Poisson ratio in drained compression (ν_C), prescribed by the *original* CLoE model. In fact, similar approach has been adopted by Mašín (2004) in the endomorphous K-hypoplastic constitutive model for clays. In this model, the bulk moduli in isotropic loading and unloading and the shear stiffness in undrained compression are calibrated. It is possible to solve the set of equations (17-22) (not detailed here) and get:

$$\nu_C = \frac{B\left(E_{ic} - E_C\right) - E_C + E_{id}}{2BE_{ic} + 2E_{id}} \quad (23)$$

with

$$B = \frac{\sqrt{3}E_C + 3E_{id}\sqrt{1+2\nu_C^2}}{\sqrt{3}E_C - 3E_{ic}\sqrt{1+2\nu_C^2}} \quad (24)$$

$$E_E = E_C\frac{1 - k\sqrt{1 + 2\left[\frac{(1+\nu_C)E_E - E_C}{E_C}\right]^2}}{1 + k\sqrt{1+2\nu_C^2}} \quad (25)$$

with

$$k = \frac{E_{ic}\left(2\nu_C - 1\right) + E_C}{3E_{ic}\sqrt{1+2\nu_C^2} - \sqrt{3}E_C} \quad (26)$$

and

$$\nu_E = \frac{E_E}{E_C}\left(1+\nu_C\right) - E_C \quad (27)$$

The Equation (23) must be solved iteratively to obtain the value of ν_C, then the Equation (25) is solved iteratively for E_E and finally we may calculate ν_E according to Equation (27).

3 Analytical formulation of basic paths

In order to fulfill the new consistency condition at isotropic stress state (Sec. 2), and to predict with reasonable accuracy the behaviour of fine grained soils, it is necessary to modify the analytical formulation of some basic paths. For conciseness' sake, the detailed description of basic paths adopted by the *original* CLoE is not presented in this paper, only modified basic paths are described. For details, the interested reader is referred to Chambon et al. (1994).

3.1 Isotropic unloading

The additional basic path, with the analytical formulation:

$$\sigma_i = \sigma_{ir} e^{\lambda_d(\varepsilon_i - \varepsilon_{ir})} \tag{28}$$

Eq. (28) introduces a new constitutive parameter λ_d, equal to E_{id}. σ_i and ε_i are the diagonal components of the isotropic stress and strain tensors respectively and σ_{ir} and ε_{ir} are their reference values. The rate formulation of the isotropic unloading path is given by (from (28)):

$$\dot{\sigma_i} = \lambda_d \sigma_i \dot{\varepsilon}_i \tag{29}$$

3.2 Triaxial drained compression - volumetric response

The initial slope of the $\varepsilon_v : \varepsilon_a$ curve of the triaxial drained compression test (where ε_v stands for volumetric strain and ε_a for axial strain) is now prescribed through the value of ν_C, linked to E_C, E_{ic} and E_{id} through Eqs. (23-24). The *original* CLoE model assumes a parabolic formulation of the $\varepsilon_v : \varepsilon_a$ curve. Because the *modified* model has the prescribed value of ν_C, the polynomial order of the formulation of the initial portion of the $\varepsilon_v : \varepsilon_a$ curve has been increased by one, in order to retain the freedom for calibration. It is now defined through the cubic polynomial equation:

$$\varepsilon_v = a_{c1}\varepsilon_a^3 + b_{c1}\varepsilon_a^2 + c_{c1}\varepsilon_a \tag{30}$$

valid for

$$\varepsilon_a \in [0, x_{pc}] \tag{31}$$

with the slope (from (30)).

$$g' = 3a_{c1}\varepsilon_a^2 + 2b_{c1}\varepsilon_a + c_{c1} \tag{32}$$

For normally compressed clays it is reasonable to assume, that at the limit surface $d\varepsilon_V$=0. Therefore, for

$$\varepsilon_a \in [x_{pc}, \infty] \tag{33}$$

we have

$$\varepsilon_v = y_{ca} \tag{34}$$

y_{ca} and x_{pc} are constitutive parameters, whose geometrical explanation is given in Fig. 1. The coefficients a_{c1}, b_{c1} and c_{c1} of the polynomial expression in (30) are calculated from y_{ca} and x_{pc} and imposed ν_C by:

$$c_{c1} = 1 - 2\nu_C \tag{35}$$

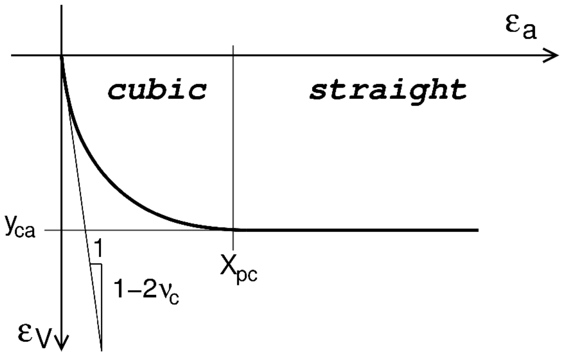
$$b_{c1} = \frac{3y_{ca} - 2c_{c1}x_{pc}}{x_{pc}^2} \tag{36}$$

$$a_{c1} = \frac{-2b_{c1}x_{pc} - c_{c1}}{3x_{pc}^2} \tag{37}$$

with the use of:

$$g'(0) = 1 - 2\nu_C \tag{38}$$

$$g'(x_{pc}) = 0 \tag{39}$$



$$\varepsilon_v(x_{pc}) = y_{ca} \tag{40}$$

Figure 1. Explanation of parameters of the triaxial compression volumetric curve

3.3 Triaxial drained extension - volumetric response

The analytical formulation of the $\varepsilon_v : \varepsilon_a$ curve of the triaxial drained extension path was modified in order to assume formulation equivalent to Eqs. (30-34). Constitutive parameters y_e and x_{m2} *are* now equivalent to y_{ca} and x_{pc} respectively.

4 Calibration procedure

Due to the consistency requirements at the isotropic stress state and at the limit stress condition, specific calibration procedure for the CLoE model has been developed (Chambon et al., 1994). The calibration procedure is slightly modified for the proposed model, in order to take into account the new basic path. In the modified procedure, the stress-strain curve of the triaxial drained compression test and isotropic loading and unloading tests are calibrated at first, in order to fix the values of the moduli *a, d* and *k* in Eqs. (2-3) at the isotropic stress state (through E_C, E_{ic} and E_{id}). Then, triaxial drained compression volumetric response and triaxial drained extension stress-strain and volumetric response may be calibrated, with prescribed ν_C, ν_E and E_E (According to Eqs. (23-27)). The proposed succesive steps for the identification procedure are as follows:

1. Limit surface
2. Triaxial drained compression stress-strain response
3. Isotropic loading test
4. Isotropic unloading test
5. Triaxial drained compression volumetric response
6. Triaxial drained extension test
7. pseudo-isotropic tests
8. shear moduli

The model has been evaluated using experiments on reconstituted normally compressed

Beaucaire clay (Costanzo et al., 2005). Calibration curves for the *original* CLoE model for the drained triaxial compression and extension tests are shown in Fig. 2. It is clear, that the model allows for a reasonable agreement between the experimental data and calibration curves. In order to calibrate the initial portion of the triaxial drained compression volumetric response accurately, however, it was necessary to underestimate the stiffness in isotropic compression, as may be seen from Fig. 3 (left). The *modified* model enables direct calibration of the moduli in isotropic loading and unloading (Fig. 3 right). The increased polynomial order of the triaxial drained compression volumetric curve (Eq. (30)) then enables calibration of this curve, which is in a reasonable agreement with experiment (Fig. 4 right). The slight discrepancy between the experimental and calibration curve could be improved by further increasing of the polynomial order of Eq. (30). External parameters of the *modified* CLoE model are summarised in Table 1.

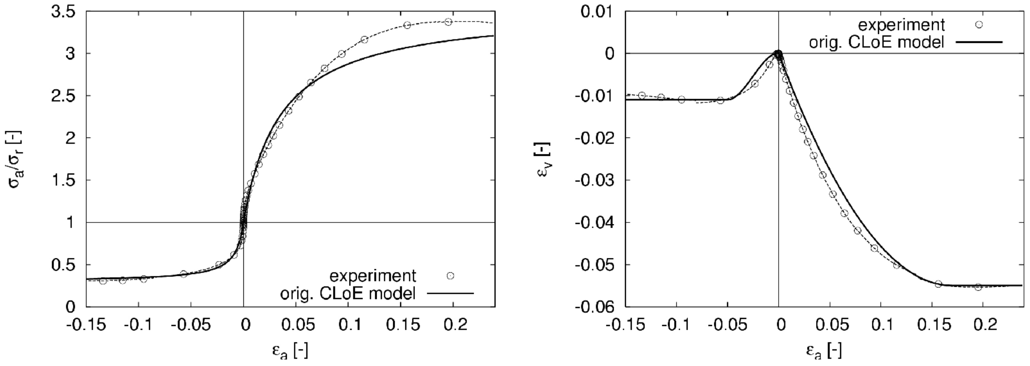


Figure 2. Calibration curves of the *original* CLoE model for the drained triaxial compression and extension tests

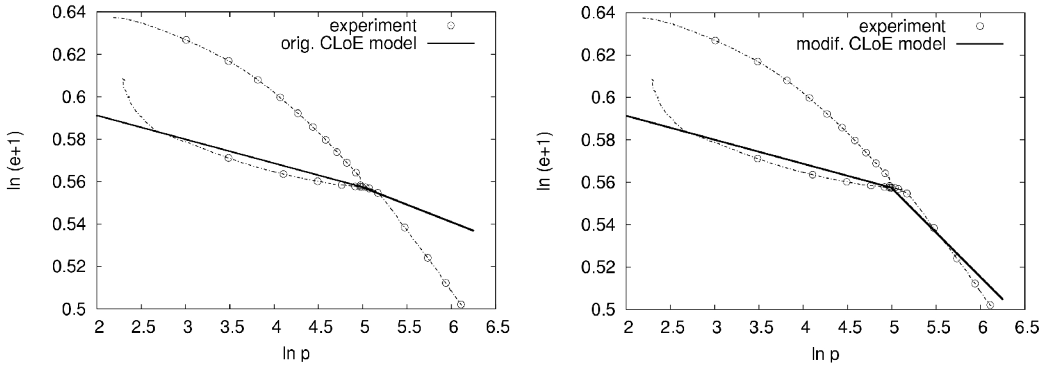


Figure 3. Calibration curves of the *modified* CLoE model for the isotropic loading test (right) compared with predictions by the *original* model (left). Note that the void ratio e is calculated from ε_v; its initial value does not influence results of simulation.

Table 1. Summary of the external parameters of the *modified* CLoE model for the Beaucaire clay.

ϕ_c	c	χ_{ca}	y_{ca}	y_{rc}	x_{pc}	p_{ref}	$\varepsilon_{v,ref}$	λ_c
34°	0 kPa	0.17	0.055	3.1	0.12	147.26 kPa	0	71.71
λ_d	ϕ_e	χ_{m2}	y_e	m_c	m_e	n	ω	
264.83	33°	-0.05	0.011	-0.2	0	-0.2	0.36	

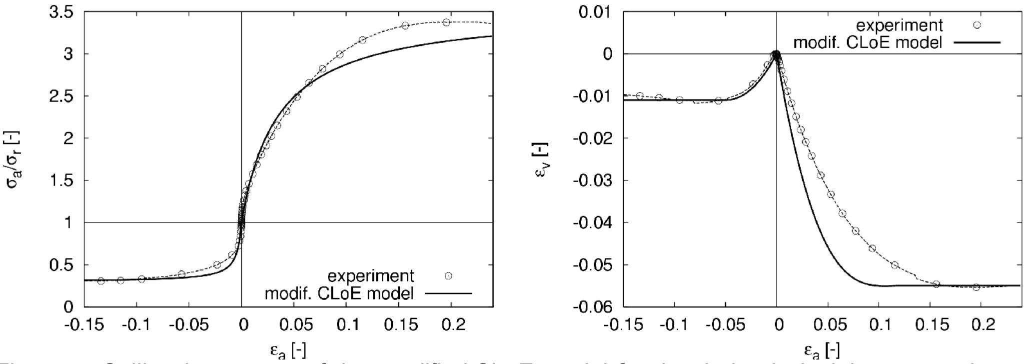


Figure 4. Calibration curves of the *modified* CLoE model for the drained triaxial compression and extension tests

5 Concluding remarks

The modification of the CLoE model presented in the paper is aimed at improving the predictive capabilities of the model for fine-grained soils. The new formulation enables a direct calibration of the isotropic loading and unloading moduli, the drained compression volumetric curve may be calibrated with a reasonable accuracy due to the increased polynomial order of its analytical formulation. In order to retain the basic features of the CLoE model, the set of state variables is restricted to the Cauchy stress. For this reason, the presented model can not be considered as suitable for problems, where larger changes in void ratio occur. The intention of the paper is rather to demonstrate the flexibility of the formulation of the CLoE model and to highlight some basic properties of hypoplastic constitutive models in general.

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