Thermo-mechanical hypoplastic interface model for fine-grained soils

Henning Stutz
Institute of Geo-Science, Marine and land geomechanics & geotechnics
Kiel University, Kiel, Germany

David Mašín
Faculty of Science,
Charles University in Prague, Prague, Czech Republic

Frank Wuttke & Boyke Prädel
Institute of Geo-Science, Marine and land geomechanics & geotechnics
Kiel University, Kiel, Germany

ABSTRACT: Thermo-active geo-structures e.g. gas-/oil-pipelines, high-voltage cables, energy-piles, nuclear waste disposals are exposed to temperature changes. These alterations have a significant effect to the behaviour of soil-structure interfaces.
The model is an adaption of the thermo-mechanical hypoplastic model from Mašín & Khalili (2011) and Mašín & Khalili (2012). The reformulation is done by redefined tensorial definitions for the special case of soil-structure interfaces. The new thermo-mechanical interface model is used for the modelling of soil-structure interface under varying temperature. By applying different temperatures and conducting a parameter study it is be proven that the model can be used for modelling various thermo-mechanical loading paths. After the simulations of some selected stress and temperature paths the paper discusses the benefits and advantages by using an advanced model for soil-structure interfaces considering temperature effects.

1 INTRODUCTION

The temperature effects in soils are manifold and can have different impacts on the behaviour of interfaces. Thermo-active geo-structures gain more and more attention in the field of renewable energy production. Typical applications as nuclear-waste storage, deep and shallow geothermal applications imply new challenges for the geotechnical & geomechanical community.
If structures are embedded into soils and the temperature fluctuates a temperature dependent interface zone is established around the embedded structure. Especially the modelling of energy pile must consider the these temperature dependences. The shaft friction and the performance of these geo-thermal foundations will be influenced by variations of the temperature. For example Di Donna & Laloui (2015), Bodas Freitas, Cruz Silva, & Bourne-Webb (2013) and Laloui, Nuth, & Vulliet (2006) emphasize the importance of the influence of the pile-soil interface for energy piles. The temperature can trigger especially in fine-grained soils changes to the mechanical behaviour. In the case of soil-interface tests the experimental available data is scarce.
Di Donna, Ferrari, & Laloui (2015) conducted sand-concrete and clay-concrete shear tests. In the sand-concrete shear test no effects of the temperature changes had been observed and measured. Whereas, in the clay-concrete tests results show an increased shear strength with respect to an increasing temperature. Further a decreasing contractive behaviour with an increase in temperature is observed. Di Donna, Ferrari, & Laloui (2015) concluded that the effect of an increasing shear stress under an increasing temperature is a result of a thermal consolidation which the sample had undergone.
Yavari, Tang, Pereira, & Hassen (2016) conducted tests with intermediate heated soils ($5-40°C$), which are typical for thermo-active geotechnical structures. The tested soil is a Kaolin clay. In contrast to the tests by Di Donna, Ferrari, & Laloui (2015) the samples were heated prior to the test which enables the tests to be performed without the effect of thermal consolidation. The results by the tests of Yavari, Tang, Pereira, & Hassen (2016) indicate negligible influ-

ences for the interface shear strength parameters. Nevertheless, Yavari, Tang, Pereira, & Hassen (2016) reported a softening behaviour for clay-concrete interfaces which are not observed in clay-clay direct shear tests.
Xiao, Suleiman, & McCartney (2014) conducted interface shear tests with a silty soil. The results were compared with the standard direct shear tests for the same soil. They concluded that the shear behaviour in soil-soil as well as in soil-solid tends to increasing shear strength under increasing temperature. Whereas, Xiao, Suleiman, & McCartney (2014) are not describing the sample preperation in detail.
From the perspective of continuum soil testing e.g. triaxial and odeometric soil testing several open questions have to be answered and should be emphasized by on-going interface research.
Mašín & Khalili (2012) hypoplastic model considers the volume change caused by heating or cooling as fully reversible process. This process is described by a constant value of the thermal expansion coefficient α_s.
The findings by (Xiao, Suleiman, & McCartney 2014, Di Donna, Ferrari, & Laloui 2015, Yavari, Tang, Pereira, & Hassen 2016) does not show the same trend. Indeed the temperature at the interface have an impact for the soil-structure interface behaviour. Due to this reason, a thermo-mechanical hypoplastic constitutive interface model is proposed. This model is based on the thermo-mechanical hypoplastic models of Mašín & Khalili (2011) and Mašín & Khalili (2012). The model reformulation by preserving the tensorial notation of the model is done by an methodology presented in Stutz & Mašín (2016) and Stutz, Mašín, & Wuttke (2016). First the thermo-mechanical hypoplastic model (Mašín & Khalili 2012) is introduced briefly and the reduced stress and strechting tensors for interface condition are given. Latter the model is used to simulate different boundary conditions which are typical for soil structure interfaces.

2 THERMO-HYPOPLASTIC INTERFACE MODEL

2.1 *Hypoplastic thermo-mechanical model in general formulation*

The thermo-mechanical hypoplastic model developed by Mašín & Khalili (2011) is introduced briefly. The stress-strain rate hypoplastic equation is given as:

$$\dot{\boldsymbol{\sigma}} = f_s \left(\boldsymbol{L} : \dot{\boldsymbol{\varepsilon}} + f_d \boldsymbol{N} \|\dot{\boldsymbol{\varepsilon}}\| \right) \tag{1}$$

where $\dot{\boldsymbol{\sigma}}$ is the stress tensor, $\boldsymbol{L}$ and $\boldsymbol{N}$ the fourth and second order constitutive tensor,$\boldsymbol{\varepsilon}$ the strain tensor and f_s and f_d the barotropy and pyknotropy factors. Mašín & Khalili (2011) developed the thermo-mechanical model using the hypoplastic clay model Mašín (2005) as basis for the improvements of the model.

The stress-stretching rate equation for the thermo-mechanical model is:

$$\dot{\boldsymbol{\sigma}} = f_s \left(\boldsymbol{L} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{TE} \right) + f_d \boldsymbol{N} \| \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{TE} \right) \| \right) + f_u H_t \tag{2}$$

The temperature related strain is given as:

$$\dot{\boldsymbol{\varepsilon}}^{TE} = \frac{\mathbf{1}}{3} \alpha_s \dot{T} \tag{3}$$

where is $\dot{T}$ the temperature rate. The constitutive fourth-order tensor L is given as:

$$\boldsymbol{L} = 3 \left(c_1 \mathcal{I} + c_2 a^2 \hat{\boldsymbol{\sigma}} \otimes \hat{\boldsymbol{\sigma}} \right) \tag{4}$$

where $\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} / \mathrm{tr} \boldsymbol{\sigma}$, the two scalars are defined as:

$$c_1 = \frac{2 \left(3 + a^2 - 2^{\alpha} \sqrt{3} \right)}{2} ; c_2 = 1 + \left(1 - c_1 \right) \frac{3}{a^2} \tag{5}$$

where r is a model parameter. The scalar value of a is defined as:

$$a = \frac{\sqrt{3} \left(3 - \sin \varphi_c \right)}{2 \sqrt{2} \varphi_c} \tag{6}$$

Where φ_c is the critical state friction angle. α is given as:

$$\alpha = \frac{1}{\ln 2} \ln \left[\frac{\lambda^* - \kappa^*}{\lambda^* + \kappa^*} \left(\frac{3 + a^2}{a \sqrt{3}} \right) \right] \tag{7}$$

Where λ^* and κ^* are model parameters. The second order constitutive tensor is then defined as:

$$\boldsymbol{N} = \boldsymbol{L} : \left(Y \frac{m}{\|m\|} \right) \tag{8}$$

where $Y = 1$ coincide with the critical stress condition of the Matsuoka–Nakai formulation. The limiting stress condition Y is defined as:

$$Y = \left(\frac{\sqrt{3} a}{3 + a^2} - 1 \right) \frac{\left(I_1 I_2 + 9 I_3 \right) \left(1 - \sin^2 \varphi_c \right)}{8 I_3 \sin^2 \varphi_c} + \frac{\sqrt{3} a}{3 + a^2} \tag{9}$$

where the stress invariants are defined as:

$$I_1 = \mathrm{tr} \left(\boldsymbol{\sigma} \right) I_2 = \frac{1}{2} \left[\boldsymbol{\sigma} : \boldsymbol{\sigma} - \left(I_1 \right)^2 \right] I_3 = \det \left(\boldsymbol{\sigma} \right) \tag{10}$$

the second order tensor m is calculated as:

$$\boldsymbol{m} = - \frac{a}{F} \left[\hat{\boldsymbol{\sigma}} + dev \hat{\boldsymbol{\sigma}} - \frac{\hat{\boldsymbol{\sigma}}}{3} \left(\frac{6 \hat{\boldsymbol{\sigma}} : \hat{\boldsymbol{\sigma}} - 1}{\left(F / a \right)^2 + \hat{\boldsymbol{\sigma}} : \hat{\boldsymbol{\sigma}}} \right) \right] \tag{11}$$

using the factor F as:

$$F=\sqrt{\frac{1}{8}\tan^2\psi+\frac{2-\tan^2\psi}{2+\sqrt{2}\tan\psi\cos 3\theta}}-\frac{1}{2\sqrt{2}}\tan\psi \tag{12}$$

with

$$\tan 3\psi=\sqrt{3}\|dev\hat{\boldsymbol{\sigma}}\| \tag{13}$$

and the Lode angle defined as:

$$\cos 3\theta=-\sqrt{6}\frac{\operatorname{tr}\left(\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}\right)}{\left[\hat{\boldsymbol{\sigma}}:\hat{\boldsymbol{\sigma}}\right]^{3/2}} \tag{14}$$

the barotropy factor is calculated as:

$$f_s=\frac{3p}{\lambda\left(T\right)}\left(3+a^2-2^{\alpha}a\sqrt{3}\right)^1 \tag{15}$$

and the pyknotropy factor:

$$f_d=\left(\frac{2p}{p_e}\right)^{\alpha} \tag{16}$$

where p_e is the Hvorslev equivalent pressure defined as:

$$p_e=p_r\exp\left[\frac{N\left(T\right)-\ln\left(1+e\right)}{\lambda^*\left(T\right)}\right] \tag{17}$$

where the reference pressure $p_r=1$kPa. The temperature dependence values of $\lambda^*\left(T\right)$ and $N\left(T\right)$ are calculated as:

$$\lambda^*\left(T\right)=\lambda^*+l_t\ln\left(\frac{T}{T_0}\right) \tag{18}$$

and

$$N\left(T\right)=N+n_t\ln\left(\frac{T}{T_0}\right) \tag{19}$$

Parameters n_t and l_t control the position and slope of the Normal Compression Line (NCL) of heated soils. The tensorial terms H_T is introduced by Mašín & Khalili (2012) to incorporate the collapse effect of the soil structure at constant effective stress for a heated soil. H_T is given by (Mašín & Khalili 2012) as:

$$H_T=-c_i\frac{\boldsymbol{\sigma}}{T\lambda^*\left(T\right)}\left[n_t-l_t\ln\frac{p_e}{p_r}\right]\langle\dot{T}\rangle \tag{20}$$

with

$$c_i=\frac{3+a^2-f_d a\sqrt{3}}{3+a^2-f_d^{SBS}a\sqrt{3}} \tag{21}$$

Where f_d^{SBS} is defined as the value of f_d at the State Boundary Surface (SBS) passing through the current stress point.

$$f_d^{SBS}=\|f_s A^{-1}\|^{-1} \tag{22}$$

The fourth order tensor $\mathcal{A}$ is expressed as:

$$\mathcal{A}=f_s\mathcal{L}-\frac{1}{\lambda^*\left(T\right)}\boldsymbol{\sigma}\otimes\mathbf{1} \tag{23}$$

The collapse behaviour is controlled by an additional factor f_u as:

$$f_u=\left(\frac{f_d}{f_d^{SBS}}\right)^{m/\alpha} \tag{24}$$

Finally, the evolution of the state variable e (void ratio) is governed by:

$$\dot{e}=\left(1+e\right)\operatorname{tr}\left(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^{TE}\right) \tag{25}$$

For an detailed description of the hypoplastic thermo-mechanical model, see Mašín & Khalili (2011) and Mašín & Khalili (2012).

2.2 *Adaption of the model for interface behaviour*

The adaption of the full thermo-mechanical hypoplastic model described in Section 2.1 is done by using reduced stress and strain rate tensors. These are defined as:

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma_n & \tau_x & \tau_z\\ \tau_x & \sigma_p & 0\\ \tau_z & 0 & \sigma_p\end{bmatrix} \tag{26}$$

and for the strain

$$\dot{\boldsymbol{\varepsilon}}=\begin{bmatrix}\dot{\varepsilon}_n & \frac{\dot{\gamma}_x}{2} & \frac{\dot{\gamma}_z}{2}\\ \frac{\dot{\gamma}_x}{2} & 0 & 0\\ \frac{\dot{\gamma}_z}{2} & 0 & 0\end{bmatrix} \tag{27}$$

These reduced tensors account for simple shear conditions at the interface can be written in modified *Voigt*-Notation as:

$$\boldsymbol{\sigma}=\begin{bmatrix}\sigma_n\\ \sigma_p\\ \tau_x\\ \tau_z\end{bmatrix}\qquad\dot{\boldsymbol{\varepsilon}}=\begin{bmatrix}\dot{\varepsilon}_n\\ 0\\ \frac{\dot{\gamma}_x}{2}\\ \frac{\dot{\gamma}_z}{2}\end{bmatrix} \tag{28}$$

By using the modifications of the standard tensorial operators used in the 3-D model from Section 2.1 and the reduced stress and stretching rate vectors the thermo-mechanical interface model is reformulated.

A detailed description of the modified tensorial notation, which is used by the stress and strain rate vector is outside of the scope of the paper. The interested reader is referred to Stutz, Mašín, & Wuttke (2016) and Stutz & Mašín (2016).
In respect to the difference in simple shear at an interface and a soil-soil shearing the influencing parameter is the surface roughness. Especially for fine-grained soils, the roughness is important. If this roughness exceed an critical value the shear failure localise in the soil specimen. Chen, Zhang, Xiao, & Li (2015) demonstrated, if the critical surface roughness is exceed the interfacial friction angle is equal to the soil-soil (internal) friction angle (Uesugi & Kishida). Due to this the roughness is for the modelling of interfaces crucial.
In hypoplastic interface models this is done e.g. Arnold & Herle (2006) by introducing a reference parameter for the roughness κ_r. Stutz & Mašín (2016) choose a different approach, by modifying r which is accounting for the shear stiffness. In clay hypoplasticity, the value of r is equal to (Mašín 2013):

$$r = \frac{4}{3}\frac{\kappa^*}{\lambda^* + \kappa^*}\frac{1+\nu}{1-2\nu} \tag{29}$$

The value of r for the reduced shear stiffness (denoted as r_r) reads

$$r_r = \frac{4/\kappa_r}{3}\frac{\kappa^*}{\lambda^* + \kappa^*}\frac{1+\nu}{1-2\nu} \tag{30}$$

where ν controls the proportion of shear and bulk stiffness and used here as a constant $\nu = 0.2$.

3 SIMULATION OF INTERFACE FRICTION BEHAVIOUR

Typically, the behaviour of interfaces is tested under different conditions. In this paper two of these boundary conditions are examined. First, the Constant–Normal–Load condition (CNL) defined as $\dot{\sigma_n} = 0$, $\dot{\varepsilon_n} \neq 0$. Secondly, the Constant-Volume condition (CV) which is defined as $\dot{\sigma}_n \neq 0$ and $\dot{\varepsilon}_n = 0$. The availability of limited number of experimental tests related to Constant-Normal-Load tests are used in conjunction to model the effects by the new thermo-mechanical interface model. Using a generic set of parameters given in Table 1. The aim of this paper is to demonstrate the application possibilities instead of comparing measurement against experimental data. The parameters given in the Table 1 are artificial parameters for the evaluation of the models response. The reference temperature is 25°C. The results of the CNL simulation are given in Figure 1 and 2 using the parameters of Soil 1. The applied normal stress is $\sigma_0 = 300$ kPa. The shear stress decreases slightly under increasing temperature, see Figure 1. Whereas, the normal strain ε_n results (see Figure 2) indicate an increasing normal strain ε_n by

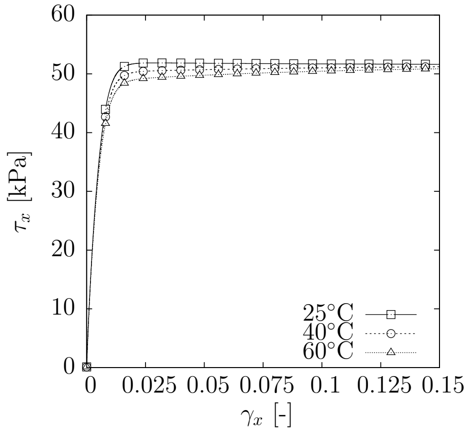

Figure 1: τ_x–γ_x results for CNL simulation with different applied constant temperatures

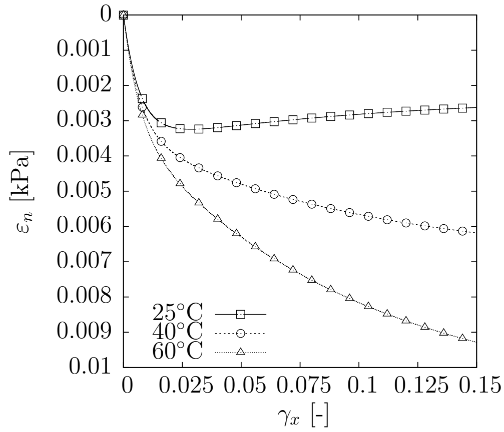

Figure 2: ε_n–γ_x results for CNL simulation with different applied constant temperatures

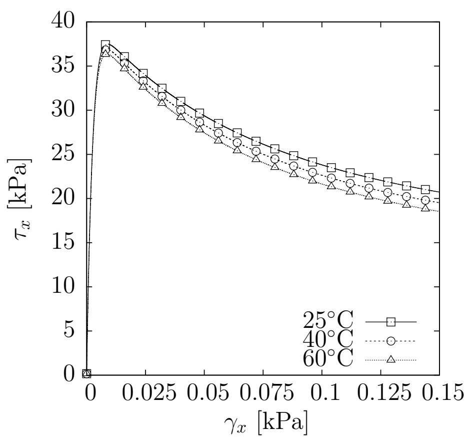

Figure 3: τ_x–γ_x results for CV simulation with different applied constant temperatures

Table 1: Parameters used for the hypoplastic thermo-mechanical interface model

Parameter	Soil 1	CV	CNL
φ_c [°]	27.5	27.5	27.5
λ^*	0.09	0.09	0.09
κ^*	0.01	0.02	0.04
N	0.88	0.82	0.82
r	0.2	0.2	0.2
α_s	$3.5 \cdot 10^{-5}$	$3.5 \cdot 10^{-5}$	$3.5 \cdot 10^{-5}$
l_t	0	var.	var.
n_t	-0.01	var.	var.
m	2.5	2.5	2.5
e_0	0.5	0.45	0.45
σ_0	300	300	100
T	var.	40	40

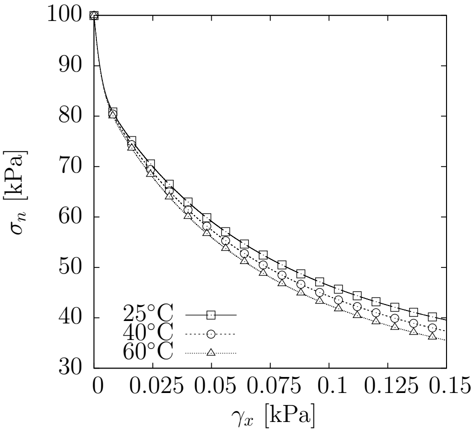

Figure 4: σ_n–γ_x results for CV simulation with different applied constant temperatures

an increasing temperature. For modelling a different behaviour as indicated by the experimental results from Di Donna, Ferrari, & Laloui (2015) another parameter set can be used, see Section 3.1.
The behaviour using a Constant Volume boundary condition is illustrated in Figure 3 and 4. Decreasing shear stresses τ_x and normal stresses σ_n are the results by increasing temperature.

In the next section the parameter variation is conducted to estimate the influence and behaviour of different model parameter which modify the thermo-mechanical behaviour of interfaces.

3.1 *Parameter variation*

Figure 5 – 8 show the results for the parameter variation of n_t and l_t under CV conditions. Those two parameters are the most important ones for modelling the thermo-mechanical interface response using the model proposed by Mašín & Khalili (2012) under constant temperature. The stress paths shown, are only monotonic shear and temperature tests.

The parameter study is conducted under constant temperature of 40°C and a normal stress of 300kPa. In all diverse figures it is observed that a positive

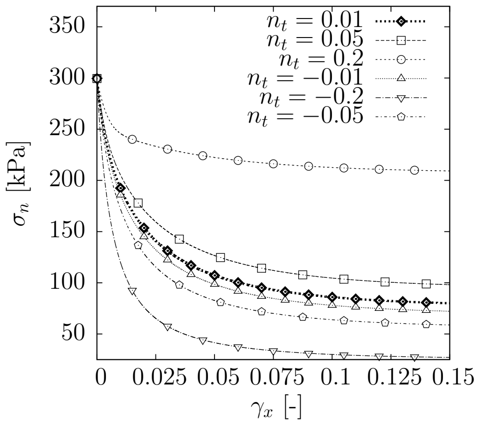

Figure 5: σ_n–γ_x results for CV simulation with parameter variation of n_t at 40°C

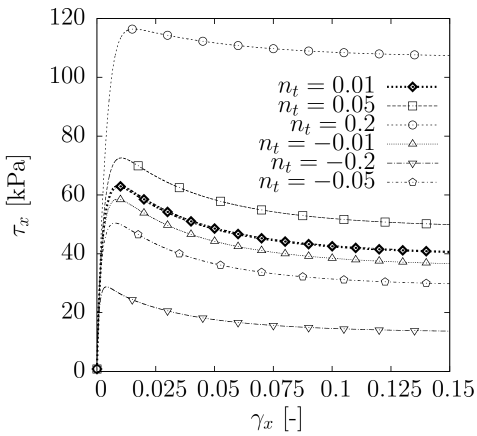

Figure 6: τ_x–γ_x results for CV simulation with parameter variation of n_t at 40°C

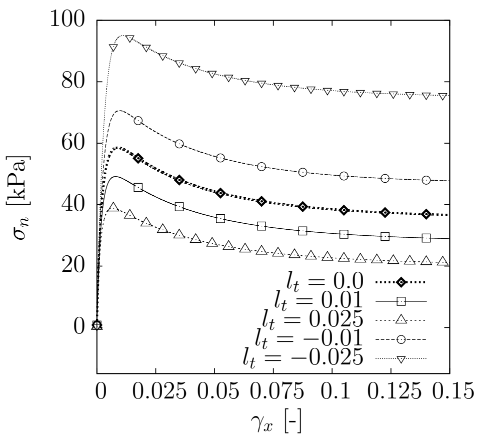

Figure 7: σ_n–γ_x results for CV simulation with parameter variation of l_t at 40°C

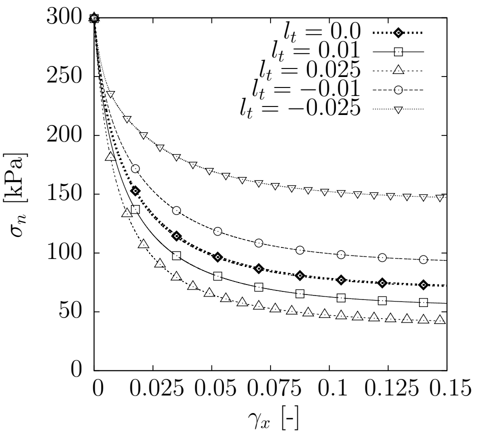

Figure 8: σ_n–γ_x results for CV simulation with parameter variation of l_t at 40°C

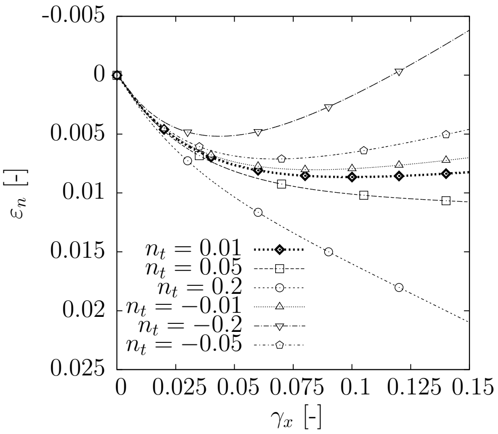

Figure 9: τ_x–γ_x results for CNL simulation with parameter variation of n_t at 40°C

sign of l_t and n_t will lead to a decrease of shear and normal stresses. Whereas, an negative sign will lead to an increasing shear stress and normal stress.

Figure 9–12 show the results for the CNL boundary conditions. The simulated tests are conducted under 100kPa. As indicated for the CV conditions the CNL results indicate the same behaviour than the CV results. In all diverse figures it is observed that a positive sign will lead to a decreasing shear stress τ and normal stress σ_n. Whereas, a negative sign will invert the model behaviour.

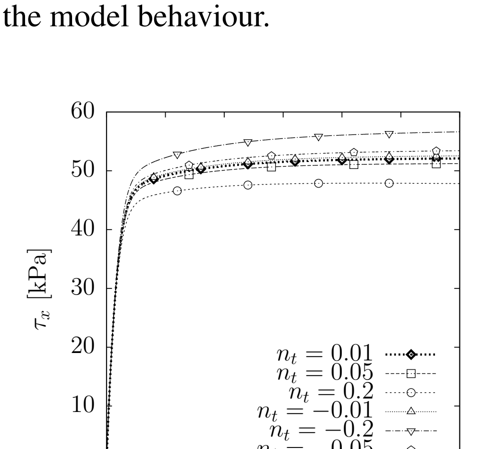

Figure 10: ε_n–γ_x results for CNL simulation with parameter variation of n_t at 40°C

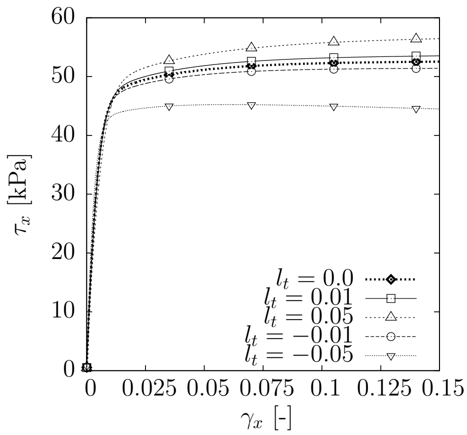

Figure 11: τ_x–γ_x results for CNL simulation with parameter variation of l_t at 40°C

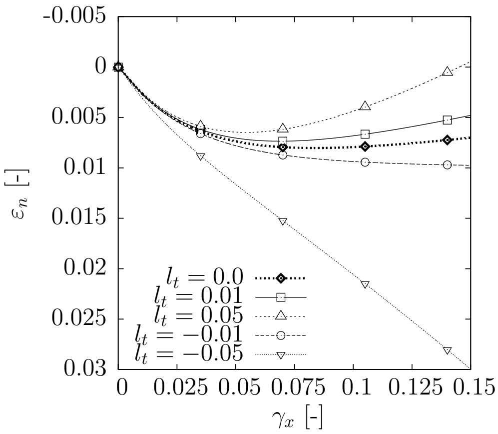

Figure 12: ε_n–γ_x results for CNL simulation with parameter variation of l_t at 40°C

4 CONCLUSIONS

In this paper we present an interface constitutive model which can take into account isothermal conditions. Further the model is capable to model non-isothermal behaviour. For this purpose a hypoplastic formulation of Mašín & Khalili (2011) and Mašín & Khalili (2012) is used and the standard tensorial notation was preserved. By the use of reduced stress and strain rate vectors in combination with modified tensorial notations described in Stutz & Mašín (2016) Stutz, Mašín, & Wuttke (2016) the interface behaviour is modelled.
The model can take into account all relevant influences which occur at an interface under thermal and mechanical loading. This is proven by the calculations of Constant-Normal-Load and Constant-Volume boundary conditions. The utilization of this model will contribute to the modelling of soil-solid interface in various conditions which are subjected to thermal and mechanical loading even for repeated loading cases.

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