Application of the fractal fragmentation model to the fill of natural shear zones

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Abstract

Statistical descriptions of the frequency size distribution of rock fragments are summarised in the introduction. It is shown, that not all soils exhibit fractal frequency size distribution. A physical process leading to the scale invariant frequency size distribution of material sheared under constrained conditions is then described from the micro-mechanical point of view. The material from natural shear zones in silty shales is examined in the framework of this theory and it is shown that this theory may be relevant. Finally some possible applications for geotechnical engineering are discussed.

1 Introduction

A variety of statistical descriptions has been used to represent the frequency–size distribution of naturally and artificially fragmented material. An extensively used empirical description is the power–law scale–invariant (fractal) relation

$$N(>r) = Cr^{-D} \tag{1}$$

where $N(>r)$ is the number of fragments with a characteristic linear dimension greater than r, D is a fractal dimension and C is a constant chosen to fit the observed distributions. An alternative empirical correlation for the size–frequency distribution is the Weibull distribution given by

$$\frac{M(<r)}{M_0} = 1 - \exp\left[-\left(\frac{r}{r_0}\right)^v\right] \tag{2}$$

where $M(<r)$ is the cumulative mass of fragments with size less than r, M_0 is the total mass of fragments and r_0 is related to their mean size [12]. The Weibull distribution is entirely equivalent to the Rosin–Rammler [7] distribution, which is extensively used in geological applications. The Weibull distribution is not scale–invariant and reduces to fractal relationship only for small fragments.

Another example of an empirical relationship describing the fragmentation process is the log–normal distribution proposed by Epstein [5].

These examples, successfully applied in many experimental studies of the frequency–size distributions of rock fragments, show that not every crushed rock (soil) granulometry may be described using fractal relationship. Following the work of Sammis and Steacy [9], physical processes which should lead to the scale–invariant distribution will be discussed theoretically in this paper and this hypothesis will be supported by examples of soil obtained from natural shear dislocations in silty shales.

2 The Micromechanics of Constrained Comminution

A theory explaining scale–invariant frequency–size distribution of the material crushed under constrained conditions has been developed by Sammis et al. [8] in order to explain observed fractal distributions of rock fragments in crustal shear zones. This theory has been discussed and supported by laboratory observations in Sammis and Steacy [9].

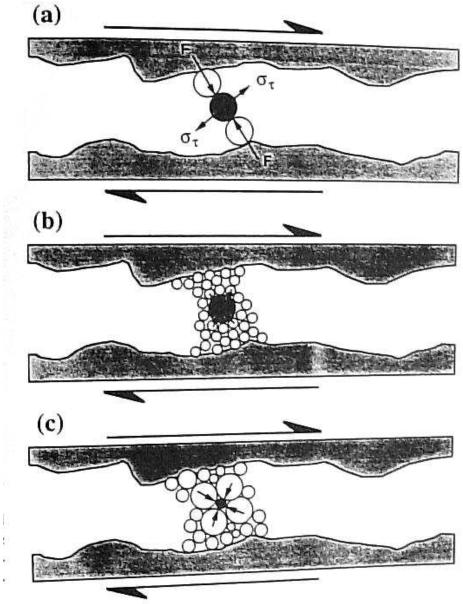

Figure 1: Schematic diagram illustrating the effect of the size of neighbouring particles on the tensile stress developed within a particle [9].

According to them frequency–size distribution is dependent on the way rock fragments are loaded. In industrial processes, such as pounding or tumbling, the volume of fragmented mass is unconstrained. Each fragment is equally likely to suffer impact, which means that the fracture probability of a given fragment is controlled by its strength. If fragment strength is independent of fragment size, then Epstein's

[5] theory is relevant and a log–normal distribution is produced. If larger fragments are more fragile then Rosin–Rammler [7] distribution is produced. In a fault zone, however, fragments are not free to change their relative positions without either additional fracturing or significant dilatational work. In this environment, a fragment's fracture probability appears to be controlled primarily by the geometry of its neighbouring fragments that supply the load.

Consider the interaction of cylindrical particles illustrated in Fig. 1. In the case (a) the shaded fragment is being loaded by two fragments of the same size. Fragments loaded this way by a compressive force F fail by tensile splitting along the load axis. In the case (b), the shaded fragment is loaded by smaller neighbouring fragments. Although the net force along the stress path is the same, components orthogonal to the stress path reduce the tensile stress. The same is true for the case (c), in which the shaded fragment is loaded by larger neighbours. This hypothesis is supported by the experimental evidence illustrated in Figure 2 [10]. The larger the percentage of 3 mm beads in the mixture, the larger the probability they are in touch with another 3 mm bead and according to the theory the larger the probability they break.

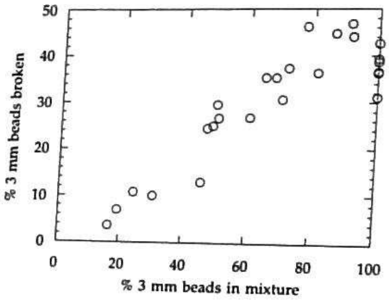

Figure 2: Percentage of 3 mm beads broken as a function of their fraction in mixtures of 3 and 1.5 mm beads [10].

If the fracture probability is maximum for neighbouring fragments of the same size (regardless their size), then the distribution evolves towards a geometry in which no two fragments of the same size neighbour at any scale. A simple fractal model, which has the same properties, is the array of cubes illustrated in Figure 3 [12]. In this model no two cubes of the same size are in contact (share face) at any scale except the smallest. The smallest cubes represent the lower fractal limit below which the power law is not valid. In the case of natural rock the lower fractal limit is delimited by the size of mineral grains. The fractal dimension D of this simple model is 2.58.

According to this theory, it is possible to explain fragmentation in the shear zone as follows: At the beginning, many large fragments have same–sized neighbours, but gradually large fragments become increasingly isolated. When there are no neighbouring large fragments (which are the most fragile due to the largest probability of

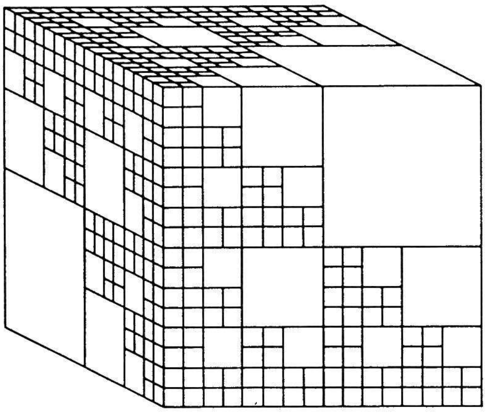

Figure 3: Fractal model for fragmentation [12].

flaws), the process shifts to the next largest size fragments for which many neighbouring pairs exist. Fragmentation process proceeds to ever smaller fragments until some fragmentation limit is reached (size of mineral grains). Then fragmentation begins again at the scale of largest fragments, since they are again the weakest, even though cushioned by smaller fragments. This continues with a cascade of fragmentation at progressively smaller scales.

The theory of constrained commituation therefore predicts fractal distribution of crushed material in the shear zone where the actual granulometry is dependent of the magnitude of tectonic movements.

3 Shear zones in the Mrázovka tunnel

The Mrázovka tunnel in Prague is designed as a part of the city highway system. The subsoil of the large diameter, two three–lane tunnels, consists mainly of clayey and silty shales of different age and weathering degree. The depth of the overburden varies from 15 to 40 meters. However, in the most critical stretch, under heavily developed urban environment the typical overburden is about 20 m. The geological site investigation revealed that tectonic shear zones intersect the tunnel profile at several locations [4]. These zones had been expected to be potential sources of unacceptable surface settlements during tunneling progress and therefore they were object of detailed investigation from the engineering–geological [3] and geotechnical [6, 1, 2] point of view.

These shear zones are usually from ten centimeters to several meters wide and they are filled with tectonically dislocated breccia–like material with varying granulometry depending on the degree of tectonical disintegration. Some shear zones are filled with smooth rock fragments "flowing" inside fine–grained matrix, in some cases the percentage of fine–grained material is smaller and the larger fragments create a skeleton. Typical small shear zone is shown in Figure 4 [3].

Figure 4: A photography of a small shear zone in the tunnel Mrázovka [3].

Mašín [6] studied the geotechnical characteristics of the fill of three shear zones – DPM1, DPM2 and DPM3 – in detail. Granulometric curves are shown in Figure 5, in which it is clear, that the proportion of fine–grained and coarse–grained material vary significantly.

According to the theory of constrained comminution, granulometry of the fill of such tectonic zones should obey fractal statistics and should be scale–invariant. The granulometric curves were re–plotted in the graph relating the number of particles larger than r as a function of their diameter r (Figure 6) using the following assumptions:

- Weight of the specimen is 10000 g, specific gravity of grains is 2.65.
- Particles have spheric shape.

It may be seen from Figure 6 that power law relationship fits reasonably well the observed data and leads to fractal dimension 2.93. This is somewhat larger than the fractal dimension predicted by the model demonstrated in Figure 3 (D=2.58). This model is therefore too simple to describe fragmentation of the fill of tectonic shear zones. The results are in accordance with Turcotte [12], who points out that fractal dimensions of fragmented rock vary significantly, but most lie in the range $2 < D < 3$.

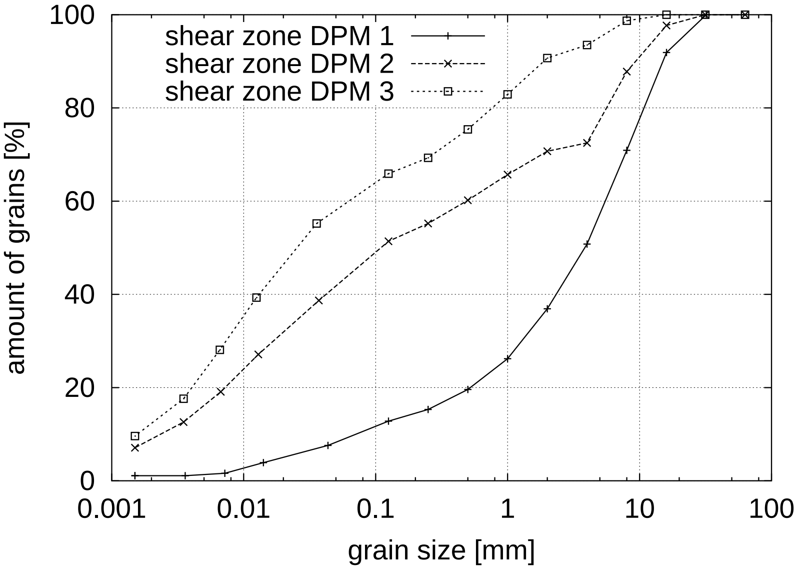

Figure 5: Granulometry of the fill from three typical shear zones in silty shales from the Mrázovka tunnel [6].

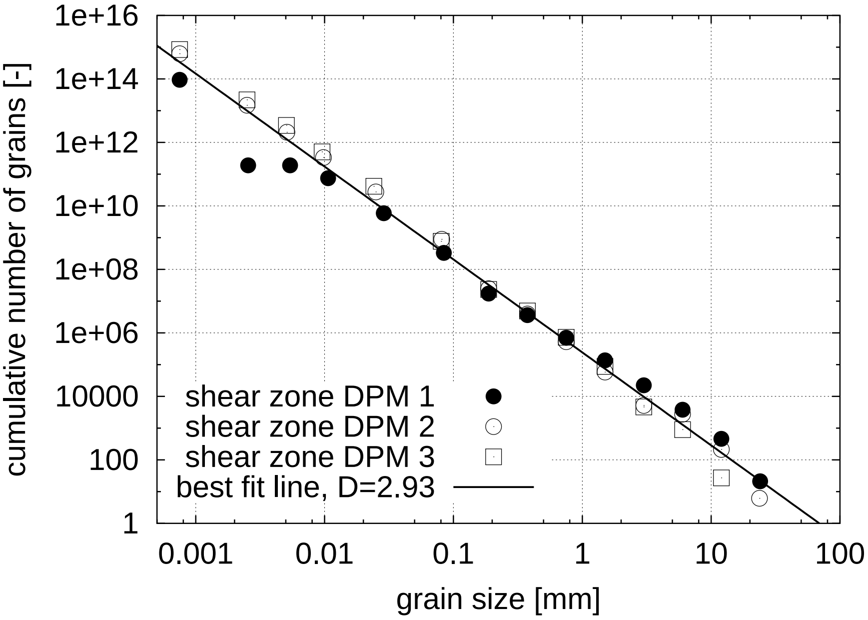

Figure 6: Number of particles larger than r as a function of their diameter r. Best fit power law relationship leads to fractal dimension D=2.93.

The constrained comminution theory can describe the variability of the grain size distribution of material from different tectonic zones. Different degrees of tectonic dislocation leads to different positions in the cascade of fragmentation described in the previous section.

4 Discussion of possible applications

In recent years the finite element method became a standard tool in predicting behaviour of various geotechnical structures. In order to perform reliable analysis, it is necessary to use constitutive models which can describe with sufficient accuracy soil behaviour. Many advanced constitutive models have been developed in past years and they have been successfully applied for analyses of different geotechnical problems. Most advanced constitutive models restrict the range of their validity either to fine grained or coarse grained soils, the behaviour of their mixtures is however not fully understood. This fact is limiting as most natural soils have gradual granulometry curve rather than being pure sands or clays.

During the investigation of the geotechnical properties of the fill of tectonic shear zones in the Mrázovka tunnel a simple theory for the behaviour of soil mixtures proposed by Thevanayagam and Mohan [11] was applied by Mašín [6] . According to this theory, it is possible to predict the behaviour of the soil mixture only on the basis of the knowledge of the behaviour of fine grained component and coarse grained component and the amount of fine grained component in the mixture. The soil should behave as a pure coarse grained component if the void ratio of coarse grained fraction (e_g) is smaller than the maximum void ratio, which the coarse grained skeleton can sustain (e_g, max). On the other hand, if grains of the coarse grained fraction "float" in the fine grained matrix and do not create a skeleton, then behaviour is governed by the fine–grained fraction. The range between these two extremes is called "transition zone". Experimentally determined dependence of the peak and critical state friction angles on the amount of fine–grained fraction in the mixture, showing this transition zone, is shown in Figure 7 [6].

Shortcoming of this theory is that granulometry curve of natural soils is gradual and it contains fractions of all sizes, therefore it is not possible to clearly distinguish between fine grained and coarse grained fraction. The second problem, which comes up, is that the size of the largest particles in the mixture is often too large and the material can not be tested in the standard laboratory equipment. Researchers are therefore forced to perform laboratory tests on material with reduced granulometry curve (with the largest particles removed). Application of results from these tests to the original material is however problematic.

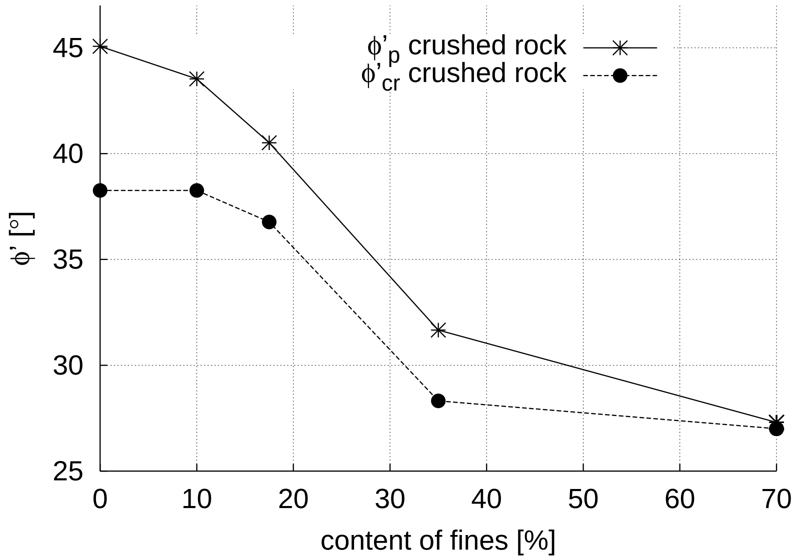

Figure 7: Dependence of the peak and critical state friction angle on the amount of fine–grained fraction in the mixture [6]

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The fact that the granulometry of (at least some) soils can be with sufficient accuracy described by fractal, scale–invariant law could be possibly used to construct more advanced conceptual model for soil mixtures and to help understanding their mechanical behaviour. Also, fractal distrubution of grain sizes in the mixture could help in evaluation of laboratory tests on material with reduced granulometry curve.

5 Conclusions

The fill material from three tectonic shear zones in silty–clayey shales has been investigated in the framework of the theory of constrained comminution, proposed by Sammis and Steacy [9]. It has been shown that this material exhibits fractal, scale–invariant distribution, and this theory is a possible micromechanical explanation of the process leading to the observed grain size distribution. Some possible applications of these observations for soils with gradual granulometric curves have been discussed.

6 Acknowledgment

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